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If both x and y are odd, is xy odd?

Answer: Yes, xy is odd.

We could just recall: an odd number times an odd number is always odd.

If you want proof, notice that odd numbers can be represented as $2m + 1$ or $2n + 1$, where m and n are integers. Multiplying two numbers of this form together would yield $4nm + 2m + 2n + 1$, which is always odd. The 1st, 2nd, and 3rd terms are multiplied by 2 (or 4), so they are even, as is their sum. An even number plus one is odd. Thus xy is odd.

When in doubt, try it out! Pick numbers to test properties.

$$\begin{array}{ccccc} 3 & \times & 5 & = & 15 \\ \text{odd} & \times & \text{odd} & = & \text{odd} \end{array} \qquad \begin{array}{ccccc} 3 & \times & 7 & = & 21 \\ \text{odd} & \times & \text{odd} & = & \text{odd} \end{array}$$

If $xy^2 < 0$, is $x < 0$?

Answer: Yes.

Any number, except for 0, raised to an even power will be positive. If y were 0, the inequality would not be true, so we know that y^2 will be positive. For xy^2 to be less than zero, x must be negative.

Simplify $\sqrt{6,300}$.

3

Answer: $30\sqrt{7}$

When simplifying an expression under the square root sign, you can factor the expression into primes. In this case, $6{,}300 = 2^2 \times 3^2 \times 5^2 \times 7$. For every pair under the square root sign, move one outside the radical, and throw the other away: $\sqrt{2^2 3^2 5^2 7}$ becomes $(2)(3)(5)\sqrt{7}$, or simply $30\sqrt{7}$.

You can also factor out perfect squares that you notice:

$$\sqrt{6{,}300} = \sqrt{63}\sqrt{100} = \sqrt{7}\sqrt{9}\sqrt{100}$$
$$= \sqrt{7} \times 3 \times 10 = 30\sqrt{7}$$

If both x and y are odd, is $x^2 + y$ odd?

Answer: No, $x^2 + y$ is even.

$x^2 = (\text{Odd})^2 = \text{Odd}$

$x^2 + y = \text{Odd} + \text{Odd} = \text{Even}$

If you want proof, notice that odd numbers can be represented as $2m + 1$ or $2n + 1$, where m and n are integers. $(2m + 1)^2 = 4m^2 + 4m + 1$, and adding $2n + 1$ would yield $4m^2 + 4m + 2n + 2$. This is always even, since a 2 can be factored from all four terms.

When in doubt, try it out! Pick numbers to test properties.

$$\begin{array}{ccccccccccccc} 3^2 & + & 5 & = & 9 & + & 5 & = & 14 \\ \text{odd}^2 & + & \text{odd} & = & \text{odd} & + & \text{odd} & = & \text{even} \end{array}$$

If x is odd and y is even, is xy odd or even?

Answer: xy is even.

xy = (Odd)(Even) = Even.

If you want proof, notice that an odd number can be represented as $2m + 1$, and an even number can be represented as $2n$, where m and n are integers. Multiplying $2m + 1$ and $2n$ would yield $4mn + 2n$, which is always even, since 2 is a factor of both terms. (Factor out the 2 to get $2(2mn + n)$, which shows that this number will be even.)

When in doubt, try it out! Pick numbers to test properties.

$$\begin{array}{ccccc} (3) & & (4) & = & 12 \\ \text{Odd} & \times & \text{Even} & = & \text{Even} \end{array}$$

If $x^{13} < 0$, is $x > 0$?

6

Answer: No.

Don't let the 13 confuse you; the only thing that matters is that 13 is an odd number. Odd exponents preserve the sign of the base. If $x^{13} < 0$, then x is also less than 0.

Simplify $\frac{85}{\sqrt{5}}$

Answer: $17\sqrt{5}$

When a square root lurks in the denominator, we can rationalize the denominator by multiplying by the appropriate form of 1—in this case, $\frac{\sqrt{5}}{\sqrt{5}}$ $\cdot \left(\frac{85}{\sqrt{5}}\right)\left(\frac{\sqrt{5}}{\sqrt{5}}\right) = \frac{85\sqrt{5}}{5}$, and 85 divided by 5 is 17, so the simplest form is $17\sqrt{5}$.

If both x and y are odd, is $x - y$ odd?

8

Answer: No, $x - y$ is even.

$x - y = \text{Odd} - \text{Odd} = \text{Even}$

If you want proof, notice that odd numbers can be represented as $2m + 1$ or $2n + 1$, where m and n are integers. Subtracting two numbers of this form would yield $(2n + 1) - (2m + 1)$, or just $2n - 2m$, which is always even, since a 2 can be factored out of the remaining terms (i.e., $2(n - m)$).

When in doubt, try it out! Pick numbers to test properties.

$$\begin{array}{ccccc} 7 & - & 3 & = & 4 \\ \text{Odd} & - & \text{Odd} & = & \text{Even} \end{array}$$

Calculate $(-1)^{789}$.

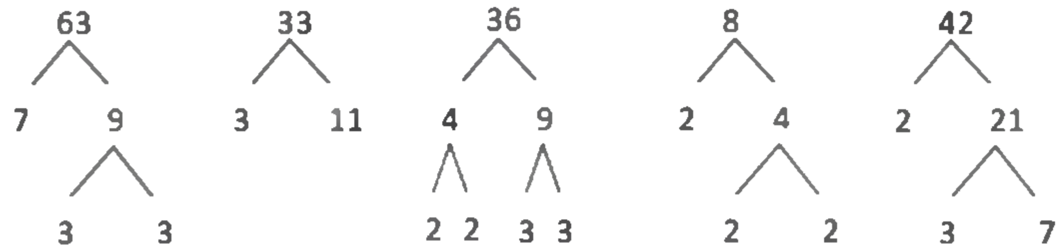
Answer: -1

Since $(-1) \times (-1) = 1$, -1 raised to any even power is 1. If you multiply by -1 one more time, you end up with -1, so -1 raised to any odd power will equal -1. 789 is an odd number, so $(-1)^{789} = -1$.

$$(-1)^{\text{odd}} = -1$$

x is divisible by 42. Which of the following numbers is definitely a factor of x^2? Choose all that apply.

[A] 63 [B] 33 [C] 36 [D] 8 [E] 42



Answer: [A] 63 and [C] 36

If x definitely has 2, 3 and 7 as factors, then when we square x, we know that x^2 will have two 2s, two 3s and two 7s as factors. 63 is $7 \times 3 \times 3$, and 36 is $2 \times 2 \times 3 \times 3$. Using the factors foundation rule, we can guarantee that all numbers that solely use those factors are factors of x^2. Both 63 and 36 use only prime factors found in x^2.

$\left(\sqrt[5]{n}\right)^5$ is always equal to which of the following?

(A) n

(B) n^{25}

(C) $n^{1/5}$

(D) 1

Answer: (A) n

Try it: $(\sqrt[5]{n})^5$ is the same as $(n^{1/5})^5$, which is equal to n, since $(n^a)^b = n^{ab}$, and 5 times 1/5 equals 1.

Alternatively, $(\sqrt[5]{n})^5 = (\sqrt[5]{n})(\sqrt[5]{n})(\sqrt[5]{n})(\sqrt[5]{n})(\sqrt[5]{n}) = n.$

If you need convincing, pick a few numbers and see what happens!

Technically, you have to be a little careful with even powers and roots (for instance, $\sqrt{x^2} = |x|$, not x by itself), but since the powers in this question are 5 and 1/5, you can apply the $(n^a)^b = n^{ab}$ rule without worry.

If both x and y are even, is $x - y$ even?

12

Answer: Yes, $x - y$ is even.

$x - y$ = Even − Even = Even.

If you want proof, notice that even numbers can be represented as $2m$ or $2n$, where m and n are integers. Subtracting two numbers of this form would yield s$2n - 2m$, or $2(n - m)$, which has 2 as a factor, so it is even.

When in doubt, try it out! Pick numbers to test properties.

8	−	6	=	2
Even	−	Even	=	Even

If $xy < 0$ and $y^2\sqrt{x} > 0$, is $y < 0$?

Answer: Yes.

If we know that $xy < 0$, then we know that x and y have different signs—one must be positive and the other negative. We know that x must be positive, because we are not allowed to take a square root of a negative number. If x is positive, then y must be negative.

If the units digit of an integer is 7, then which one-digit integers is it definitely NOT divisible by?

Answer: 2, 4, 5, 6, and 8

Integers that are divisible by 2, 4, 6, or 8 end in 2, 4, 6, 8, or 0; those divisible by 5 end in 5 or 0.

As an exercise, try to provide examples of integers with a ones digit of 7 that are divisible by 1, 3, 7, and 9.

Calculate $16^{5/4}$.

Answer: 32

Using the rules of exponents, $16^{5/4} = (16^5)^{1/4} = \left(16^{1/4}\right)^5$ Since it is easier to calculate $16^{1/4}$ than it is to calculate 16^5, the latter representation will be easier to simplify. $16^{1/4} = \sqrt[4]{16} = 2$ (because $2^4 = 16$), and $2^5 = 32$.

If x is divisible by y, and both x and y are odd, could $\frac{x}{y}$ be odd, even, either, or neither?

16

Answer: $\frac{x}{y}$ must be odd.

This question is tricky, because an odd divided by an odd can yield an odd integer *or a non-integer*. However, the question states that x is divisible by y. Therefore, x/y is an integer, and the result must be odd.

To see why x/y can never be even, consider the following information. If x is odd, then all its factors are odd. So even when x is divided by y, the factors that remain will all still have to be odd.

If an integer that is divisible by 6 is squared, then which (nonzero) one-digit integers is this squared result definitely divisible by?

Answer: 1, 2, 3, 4, 6, and 9

Call the original integer n. Since n is divisible by 6, we can say $n = 6m$, where m is any integer. Squaring n yields $n^2 = (6m)^2 = 36m^2$. Since 36 is divisible by 1, 2, 3, 4, 6, and 9, they are all factors of n^2 as well.

Any combination of 5, 7, and/or 8 may also divide n^2, but we can't say for sure whether they do without knowing what m is.

$$\frac{(6^4)(50^3)}{(2^4)(3^4)(10^3)} =$$

Answer: 5^3, or 125

Instead of multiplying out everything, look for ways to reduce. On the top of the fraction, 6^4 can be separated into $(2^4)(3^4)$. This can be cancelled with the 2^4 and 3^4 on the bottom of the fraction, so we are left with $\frac{50^3}{10^3}$, which can be reduced to 5^3, which equals $5 \times 5 \times 5 = 125$.

Simplify the following expression: $(4(6(8(9^0))^1)^{-1})^2$

19

Answer: 1/144

PEMDAS dictates the order of operations to perform. We must always calculate the innermost parentheses first, then work our way outwards. Calculate $9^0 = 1$ first; then $8^1 = 8$. Next, $(6(8))^{-1} = 1/48$; then $(4/48)^2 = (1/12)^2 = 1/144$.

It's easy to remember PEMDAS with this saying: Please Excuse My Dear Aunt Sally!

If the units digits of an integer is 0, then which nonzero one-digit integers is the integer definitely NOT divisible by, if any?

Answer: None

It could be divisible by <u>any</u> of the one-digit integers! (Except for 0; dividing by 0 is always off limits.)

To verify, take any nonzero one-digit integer, multiply it by ten, and the product will end in zero and be divisible by the original one-digit integer.

70 is divisible by 7.
90 is divisible by 9. Etc.

If x is even and y is odd, is $x^2 + y^2$ even or odd?

Answer: $x^2 + y^2$ is odd

$x^2 + y^2 = (\text{Even})^2 + (\text{Odd})^2 = \text{Even} + \text{Odd} = \text{Odd}.$

If you want proof, notice that an even number can be represented as $2m$, and an odd number can be represented as $2n + 1$, where m and n are integers. Squaring the even number yields $4m^2$; the odd, $4n^2 + 4n + 1$. Adding these together yields $4m^2 + 4n^2 + 4n + 1$. The 1st 3 terms all have 4 as a factor, so their sum is even, and an even number plus 1 is odd.

When in doubt, try it out! Pick numbers to test properties.

2^2	+	3^2	=	4	+	9	=	13
$(\text{Even})^2$	+	$(\text{Odd})^2$	=	Even	+	Odd	=	Odd

If $y^7 < y^6$, describe all of the possible values for y.

Answer: $y < 1$, but not equal to 0 (alternatively, $0 < y < 1$ or $y < 0$).

Think about various categories of numbers: if y were negative, then y^7 would also be negative, while y^6 would be positive; then $y^7 < y^6$. If $y = 0$ or 1, then $y^7 = y^6$, which is not acceptable. When y is between 0 and 1, $y^7 < y^6$, since y^7 would equal y^6 times some fraction between 0 and 1. Finally, when $y > 1$, $y^7 > y^6$.

When in doubt, try it out! For instance, if $y = -1$, then $y^7 = (-1)^7 = -1$. $y^6 = (-1)^6 = 1$. y^7 is in fact less than y^6. You can also test $-1/2$, $1/2$, 1, and 2 to see the pattern.

If the positive integer x is a prime number and $x + 11$ is also a prime number, what is x?

23

Answer: $x = 2$

If you tested numbers to answer this question, you probably figured out pretty quickly that 2 is a possible value of x. If you continue to test numbers to make sure there are no other possible values for x, you may notice a pattern emerging. $11 + 3 = 14$, $11 + 5 = 16$, $11 + 7 = 18$, etc. 11 plus any prime besides 2 will yield an even number. 2 is the only even prime, because every other even number has 2 as a factor. Therefore, x must equal 2.

If $a^b < 0$, which of the following could be true? Select all that apply.

[A] $a < 0$

[B] $a > 0$

[C] $b = 0$

[D] b is even

[E] b is odd

Answer: [A] and [E]

In order for $a^b < 0$, ***a* must be negative**. (This is equivalent to saying that $a < 0$.) A positive base to any power (even a negative power) will be positive.

Furthermore, ***b* cannot be even**, as an even exponent "hides the sign" of the base. In contrast, odd exponents always keep the original sign of the base.

Finally, could $b = 0$? Any nonzero base raised to the 0 power yields 1, and 0^0 is not defined. Thus, if b were 0, a^b would equal 1, (if it equals a number at all), which is positive. Therefore, ***b* cannot equal 0**.

To prove that ***a* could be negative** and ***b* could be odd**, consider these examples: $(-3)^3 = -27$ and .

If both x and y are even, could $\frac{x}{y}$ be odd, even, either, or neither?

Answer: $\frac{x}{y}$ could be either even or odd, and could also be neither (that is, a non-integer).

Even numbers can be represented as $2m$ or $2n$, where m and n are integers. (Think about why this is.) Dividing would give $(2n)/(2m)$, or just n/m. This ratio could be even or odd, and might not be an integer at all!

For example, $\frac{x}{y}$ could be even: $\frac{40}{4}=10$

$\frac{x}{y}$ could be odd: $\frac{44}{4}=11$

$\frac{x}{y}$ could be a non-integer: $\frac{42}{4}=10.5$

What is the greatest number of primes that could be included in a set composed of four consecutive positive integers? Name the elements of the set.

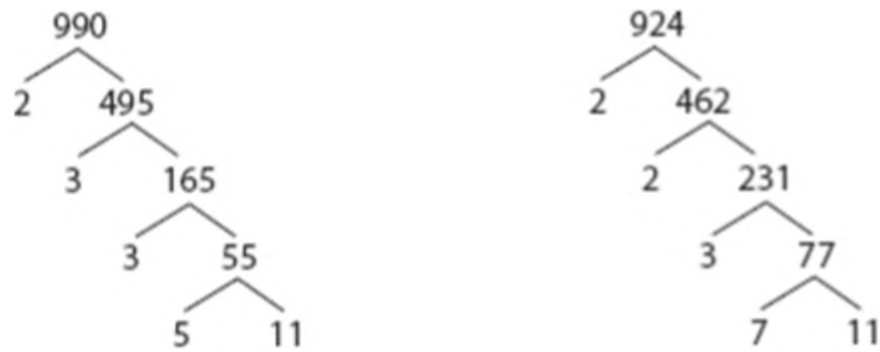
Answer: 3, in the set {2, 3, 4, 5}

Any set composed of four consecutive positive integers will contain two even and two odd integers. Since 2 is the only even integer that is prime, no such sets can have four primes, and sets that do not contain 2 can have, at most, two primes. The only set with three primes is {2, 3, 4, 5}.

Why isn't {1, 2, 3, 4} acceptable as another solution to this question?

Answer: The number 1 is not prime!

What is the greatest common factor of 990 and 924?



Answer: 66

To find the Greatest Common Factor of 2 or more numbers, figure out all the prime factors they share in common. In this case, 990 and 924 each have one 2, one 3 and one 11. That means that the GCF will be $2 \times 3 \times 11$, or 66.

x is divisible by 144. If $\sqrt[3]{x}$ is an integer, then which of the following is $\sqrt[3]{x}$ definitely divisible by? (Choose all that apply)

[A] 4

[B] 8

[C] 9

[D] 12

[E] 16

144

2, 2, 2, 2, 3, 3

Answer: [A] 4 and [D] 12

Remember that when we complete a prime box for a variable, that variable could still have additional factors. For the cube root of a number to be an integer, the original number must have 3 copies of each prime factor in its prime box, or some multiple of 3 (3, 6, 9, etc.). In this case, that means the factors of x that we can't see must include at least two additional 2s and one additional 3. From this information, we can definitively conclude that $\sqrt[3]{x}$ must have two 2s and one 3 as factors. 4 (= 2 × 2) and 12 (= 2 × 2 × 3)are the only numbers in the list we can guarantee are factors of $\sqrt[3]{x}$.

What is the only two-digit number that is both a perfect square and a perfect cube?

Answer: 64

We need a 2-digit integer that is both a perfect square *and* a perfect cube. This set includes all integers of the form $m^3 = n^2$, where both m and n are integers. Manipulating the equation tells us that $n = m^{3/2}$. Thus we can only choose integers for m that will make n an integer—so m must be a perfect square. The only perfect square that works is 4: $4^3 = 64$, a 2-digit integer. 9 doesn't work, because $9^3 = 729$, a 3-digit integer. 1 doesn't work either, because $1^3 = 1$, a 1-digit integer.

Alternatively, you can pick numbers. Figure out all the 2-digit cubes and then see which is also a perfect square:

$1^3 = 1$ $\quad 2^3 = 8$ $\quad 3^3 = 27$ $\quad 4^3 = 64$ $\quad 5^3 = 125$

Of the numbers in this list, 5^3 is too big. Only 64 is also a perfect square (8^2).

If $x - y > 0$, is $|x| > |y|$?

Answer: Maybe.

When variables are inside absolute values, a big unknown is whether the variables are positive or negative. If x and y are both positive, then the answer to the question will be yes.

But now suppose that x is 3 and y is -7. $3 - (-7) = 10$. In this case, the answer to the question is no, since $|3|$ is less than, not greater than, $|-7|$.

Dinner cost \$230 including a 15% tip. How much did dinner cost without the tip?

Answer: $200

If $230 includes the cost of the dinner plus an additional 15%, then it is 115% of the cost of the dinner. Let *x* be the cost of the dinner:

$$230 = 1.15x$$

$$230 = \frac{115x}{110}$$

$$\frac{100}{\cancelto{1}{115}} \times \cancelto{2}{230} = \frac{\cancelto{1}{115}x}{\cancelto{1}{100}} \times \frac{\cancelto{1}{100}}{\cancelto{1}{115}}$$

$$200 = x$$

Alternatively, guess the cost of the dinner, add 15% and see how close you get to $230. $200 should be a natural guess, and $200+.

$$(10^4)(0.000001) =$$

Answer: 0.01 or 10^{-2}

An easy shortcut when dealing with powers of 10 is to simply move the decimal over the same number of units as the exponent. In this case, the exponent is 4, so we move the decimal to the right 4 places. Alternatively, 0.000001 can be rewritten as 10^{-6}, and $(10^4)(10^{-6}) = 10^{-2}$.

Which number is closest to 7% of 1,440?

(A) 50

(B) 75

(C) 100

Answer: (C) 100

We can save time by estimating. 1,440 is approximately 1,400, which is 14 × 100.
7% of
$$(14)(100) = (7/\cancel{100})(14)(\cancel{100}) = 7 \times 14 = 98.$$
This is a slight underestimate, so answer choice C) must be correct.

Alternately, we know that 10% of 1,440 is 144, and 5% is half that, or 72.

75 is too close to 72, so the only answer that makes sense is 100.

The original price of an iPhone® was increased by 25%. A sale brought the price of the iPhone® back down to its original price. The sale reduced the new price of the iPhone® by what percent?

Answer: 20%

Start with a smart number. Assume the price of the iPhone® is $100. 25% of 100 is 25, so the increased price was $125. We know the sale then reduced the price of the phone to its original price, $100, so the sale reduced the price by $25, because 125 – 100 = 25. The percent decrease is the difference in prices divided by the original price. 25/125 reduces to 1/5, which is 20%.

Which fraction is greater in each pair?

Answer: $\frac{5}{8}$ and $\frac{89}{170}$

For the first set of fractions, cross multiply and compare the numerators. $50\ \frac{5}{8} \qquad \frac{6}{10}\ 48$ 50 is greater than 48, so $\frac{5}{8}$ is greater.	For the second set of fractions, estimate. $\frac{132}{300}$ is less than half, whereas $\frac{89}{170}$ is more than half. $\frac{89}{170}$ is thus larger.

$$\frac{1,863,471}{626,502} \approx$$

(A) 3

(B) 4

(C) 5

(D) 30

(E) 35

Answer: (A) 3

We are only asked for an approximate answer, so use the heavy division shortcut.

$$\frac{1{,}8\cancel{6}\cancel{3},\cancel{4}\cancel{7}\cancel{1}}{6\cancel{2}\cancel{6},\cancel{8}\cancel{0}\cancel{2}} \approx \frac{18}{6} \approx 3$$

A bag of jellybeans contains 4 flavors: watermelon, cherry, orange and pear. 1/4 of the jellybeans are watermelon, 1/3 are cherry, 1/6 are orange, and the rest are pear. What percent of the jellybeans are pear?

Answer: 25%

First we need to find out what fraction of the jellybeans are not pear flavored. We have to add the fractional amounts of the other flavors. The common denominator is 12, so $\frac{3}{12}+\frac{4}{12}+\frac{2}{12}=\frac{9}{12}=\frac{3}{4}$. Thus, $1 - 3/4 = 1/4$ of the jellybeans must be pear. 1/4 expressed as a percent is 25%.

What is 35% of 120?

Answer: 42

Although 35% of a number is not easy to find without some calculation, 10% and 5% are usually easier.

$35\% = 3 \times 10\% + 5\%$

10% of 120 is 12 and 5% is half of 10%, so 5% of 120 is 6.

$3 \times (12) + 6 = 36 + 6 = 42$

What is the units digit of $(2^7)(7^4)(5^6)$?

Answer: 0

Although you could multiply everything out, that is too time-consuming. Notice that $2 \times 5 = 10$. That means the units digit is 0. Anything multiplied by 0 is 0, so we know that the units digit of the final product will be 0.

Last year, John earned a combined $147,000 from his salary and bonus, and his bonus was equal to half of his salary. What was John's bonus last year?

Answer: $49,000

If John's bonus was equal to half of his salary, then John's salary was twice his bonus. His total income was 1 bonus plus the equivalent of 2 bonuses, or total income was 3 times his bonus. His bonus was thus $49,000.

This could also be shown algebraically:

$S + B = 147{,}000$

$S = 2B$

Substituting: $S + B = (2B) + B = 3B = 147{,}000$

So, $B = \frac{147{,}000}{3} = 49{,}000$. Notice that $150 \div 3 = 50$, so $147 \div 3 = 49$.

21,267 is approximately what percent of 10^6?

(A) 0.2%

(B) 2%

(C) 20%

Answer: (B) 2%

Use benchmark values to estimate. $10^6 = 1{,}000{,}000$. Finding 1% is the same as dividing by 100, so 1% of 10^6 is 10^4 or 10,000.

21,267 is a little more than twice 10,000, so 21,267 is approximately 2% of 10^6. You could also use division to estimate your answer:

$$\frac{21{,}267}{10^6} = \frac{2\cancel{1},\cancel{267}}{1{,}00\cancel{0},\cancel{000}} \approx 2\%$$

Which of the following is closest to 23% of 41/60 of 240 rounded to the nearest integer?

(A) 24 (B) 39 (C) 52 (D) 68

Answer: (B) 39

The answer choices are far apart, so we can save time by estimating. 41/60 is close to 40/60, which is 2/3. $240 \times 2/3 = 160$. 23% is close to 25%. To calculate 25% of a number, just divide by 4. 160/4 is 40. The best answer is (B) 39.

$$\frac{(57)(10^3)(0.001)}{(10^4)(10^{-2})} =$$

Answer: 0.57

First, change 0.001 to 10^{-3}. Now, combine the terms on the top and the bottom.

$(10^3)(10^{-3}) = 10^0 = 1$

$(10^4)(10^{-2}) = 10^2$

We are left with $\frac{(57)(1)}{(10^2)}$. To divide by 10^2, just move the decimal to the left 2 places. 57 becomes 0.57.

The combined revenue for a company for last year and this year was $700,000. If revenue decreased 40% from last year to this year, what percent of the combined revenue was earned last year?

Answer: 62.5%

The $700,000 amount is actually irrelevant; the answer would be the same even if combined revenue were a different amount.

x = revenue last year, so revenue this year = (x – 40% of x) = 0.6x

Thus, combined revenue = x + 0.6x = 1.6x

Percent of the combined revenue that was earned last year = $\left(\frac{x}{1.6x}\times 100\right)\% = \left(\frac{100}{1.6}\right)\% = 62.5\%$

The price of a television increased from \$180 to \$216. What is the percent increase in the price?

Answer: 20%

Percent change is equal to change divided by original value. The change is 216 − 180 = $36. The original price is $180. 36/180 reduces to 1/5, which is the same as 20%.

What percent of 1.5×10^7 is 4,500,000?

46

Answer: 30%

$1.5 \times 10^7 = 15{,}000{,}000$. We can use benchmark values to estimate. 10% of 15,000,000 is 1,500,000. This is too small. But notice that 1,500,000 is 1/3 of 4,500,000, so if we triple 10% of 15,000,000, we'll have our answer.

Therefore, 4,500,000 is 30% of 1.5×10^7.

If x is a prime number, what could NOT be the units digit of 7^x? Select all that apply.

[A] 1

[B] 3

[C] 7

[D] 9

Answer: [A]

When trying to find the units digit of a number, ignore all the other digits in the number. $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2{,}401$, $7^5 = 16{,}807$, $7^6 = 117{,}649$, etc. The units digit of 7 raised to the first few powers is 7, 9, 3, 1, 7, 9, 3, 1, etc. The pattern repeats: [3, 9, 7, 1]. Since x is prime, it can only be even when $x = 2$. This implies that x cannot be a multiple of 4, and the units digit of 7^x cannot be 1, the 4^{th} term in the units digit pattern. All of the other choices are possible, as $7^2 = 49$, $7^3 = 343$, and $7^5 = 16{,}807$ prove.

The price of a refrigerator is increased by 50%. It then goes on sale, with the new sale price equaling 75% of the *original* price. The sale price is what percent of the *increased* price?

Answer: 50%

When solving word problems involving percents, it's usually helpful to pick 100 as your starting value. If the price is increased by 50%, the new price is $150. The sale reduces the price to 75% of the *original* price. $100 is the original price, so the sale reduces the price to $75. The question asks what percent the sale price is of the *increased* price. 75/150 = 1/2 = 50%.

What is the units digit of $(5^3)(7^2)(3^2)$?

Answer: 5

When solving for the units digit of a number, you can ignore all the other digits.

$5^3 = 125$. Drop the other digits and keep the 5.

$7^2 = 49$. Drop the other digits and keep the 9.

$3^2 = 9$. Keep the 9.

$5 \times 9 = 45$. Keep the 5.

$5 \times 9 = 45$.

The units digit is 5.

Carla earns a base salary plus 10% commission on her total sales revenue exceeding $50,000. If her total sales revenue had been 20% greater, her commission would have been 25% greater. How much did she actually make on commission this year?

Answer: \$20,000

Label Carla's commission c and her total sales revenue r.

From the first sentence, $c = 0.1(r - 50{,}000)$

From the second sentence, $1.25c = 0.1(1.2r - 50{,}000)$.

Multiply both equations by 10 to eliminate the 0.1 term.

First: $10c = r - 50{,}000$

Second: $12.5c = 1.2r - 50{,}000$

Solve the first equation for r and substitute into the second equation, then solve.

First: $r = 10c + 50{,}000$

$$\begin{aligned}\text{Second: } 12.5c &= 1.2(10c + 50{,}000) - 50{,}000\\ 12.5c &= 12c + 60{,}000 - 50{,}000\\ 0.5c &= 10{,}000\\ c &= \$20{,}000\end{aligned}$$

Factor:

$$x^2 - 11x + 30 = 0$$

Answer: $(x-5)(x-6)=0$

Since the last sign is positive, set up 2 parentheses with the sign of the middle term.

$(x-\;)(x-\;)$

Find two numbers that multiply to 30 and add to 11 and place them in the parentheses.

$(x-5)(x-6)$

What values for x solve the equation?

For each of the following, could the answer be an integer if x is an integer greater than 1?

(A) $x^{10} + x^{-10} =$

(B) $x^{1/6} + x^{1/2} =$

Answer: (A) No; (B) Yes

(A) **No**. $x^{-10} = 1/x^{10}$. For any $x > 1$, this won't be an integer.

(B) **Yes**. This is equivalent to $\sqrt[6]{x} + \sqrt{x}$, so if x has an integer sixth root this will be an integer. For example, if x equals 64, the sixth root of x is 2, and the square root is 8.

Any number with an integer sixth root will have an integer square root. Why?

Is it possible to solve for a single value of x in each of the following systems of equations?

(A) $2x + 3y = 8$

$2x - y = 0$

(B) $x^2 + y - 24 = 0$

$y = 2x$

(C) $2x - 4y = 13$

$-6x + 12y = -39$

Answer: (A) Yes; (B) No; (C) No

(A) **Yes**. We are given 2 linear equations. There are no xy terms or x/y terms.

(B) **No**. There is an x^2 term. If you substitute $2x$ in for y, you can factor the equation to $(x + 6)(x - 4) = 0$, which produces two possible values for x (namely, 6 and 4).

(C) **No**. The two equations are equivalent. The second equation is just the first equation multiplied by -3.

What is S_{25} in the following sequence?

$S_n = S_{n-1} - 10$ and $S_3 = 0$.

Answer: −220

First, we need to convert the recursive sequence definition provided into a direct sequence formula. Each term is 10 less than the previous one. Therefore $S_n = -10n + k$, where k is some constant that we must determine. Use S_3 to find a value for k: $0 = -10(3) + k$. Thus, $k = 30$, so $S_n = -10n + 30$. Now we plug in 25 for n: $S_{25} = -10(25) + 30 = -220$.

Alternatively, we could plug in 0 for S_3 and find that $S_4 = -10$, $S_5 = -20$, $S_6 = -30$, etc. This pattern is that the subscript (4, 5, 6) is 3 more than the tens digit (1, 2, 3). Thus, $S_{25} = -220$.

If -5 is one solution to the equation $x^2 + kx - 10 = 0$, where k is a constant, what is the other solution?

Answer: $x = 2$

If one solution is -5, we know one of the factors of the quadratic expression is $(x + 5)$. We now know the other factor is $(x - 2)$ because the two numbers in parentheses must multiply to -10. Therefore the other solution is $x = 2$.

Bonus: What is k?

From the solution above, if $(x + 5)(x - 2) = 0$, then distributing yields $x^2 + 3x - 10 = 0$. Thus, $k = 3$.

Alternatively, plug $x = -5$ into the original quadratic and solve for k. $(-5)^2 + k(-5) - 10 = 0$ becomes $25 - 5k - 10 = 0$, or $15 = 5k$. Again, $k = 3$.

If $ax < ay$, is $x < y$?

Answer: Maybe (it depends on the sign of a)

It may be tempting to simply divide by a on both sides. However, this will only yield $x < y$ if a is positive. Remember that when you multiply or divide an inequality by a negative number, you must flip the sign. So if a is negative, the conclusion would be that $x > y$. Depending on the sign of a, the answer could be yes or no.

Solve for y:

$$y^2 + 7y - 60 = 0$$

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Answer: $y = -12, 5$

Since the last sign is negative, set up 2 parentheses with opposite signs. $(y + \ )(y - \ )$

Find two numbers that multiply to 60 and differ by:

$12 \times 5 = 60 \qquad 12 - 5 = 7$

Place the larger number in the parentheses with the same sign as the middle term ($+7y$):

$(y + 12)(y - 5) = 0$

If $y + 12 = 0$, then $y = -12$. If $y - 5 = 0$, then $y = 5$.

What is the value of *x*?

$$5^{3x} = 5^{7x - 4}$$

Answer: 1

Since the bases are equal, we can simply set the exponents equal to each other.

$$3x = 7x - 4$$
$$4 = 4x$$
$$1 = x$$

What is the minimum value of $f(x) = -5 + (x + 7)^2$, and at what value of x does it occur?

Answer: minimum value = −5, $x = -7$

The squared expression will always be non-negative, so to make $f(x)$ as small as possible, make the squared expression as small as possible—set it equal to zero. If $x + 7 = 0$, $x = -7$. Once you have the x value, plug it back into the original equation to solve for the minimum value. $f(x) = -5 + (0)^2$. Therefore, the minimum value is −5.

Remember, $f(x)$ and y are synonymous.

What are all possible values of x?

$$x^2 - 27x + 50 = 0$$

Answer: $x = 2$ or 25

Since the last sign is positive, set up 2 parentheses with the sign of the middle term.

$(x - \ )\ (x - \ )$

Find two numbers that multiply to 50 and add to 27 and place them in the parentheses.

$(x - 2)\ (x - 25) = 0.$

If $x - 2 = 0$, then $x = 2$. If $x - 25 = 0$, then $x = 25$.

Solve for b:

$$-\frac{b}{7} \geq 4$$

61

Answer: $b \leq -28$

To isolate b, multiply both sides by -7 and flip the direction of the inequality sign.

When multiplying or dividing an inequality by a negative number, remember to switch the direction of the inequality sign.

Use factoring to simplify the following expressions:

(A) $4^5 + 4^5 + 4^5 + 4^5$

(B) $xw + yw + zx + zy$

Answer: (A) 4^6; (B) $(w + z)(x + y)$

(A) The greatest common factor is 4^5.

$$4^5(1 + 1 + 1 + 1) = 4^5(4) = 4^6.$$

Make sure to look for common terms that can be factored out of an expression. Factoring is often a crucial step toward solving an equation.

(B) Factor by grouping: $(xw + yw) + (zx + zy) =$

$$w(x + y) + z(x + y) = (w + z)(x + y).$$

If you have 4 terms and 4 variables, look to factor by grouping.

Solve for each of the following:

(A) If $x = \frac{7 - y}{2}$, what is $2x + y$?

(B) If $\sqrt{2t + r} = 5$, what is $3r + 6t$?

Answer: (A) 7; (B) 75

(A) Multiply both sides by 2 and add y to each side.

$$2(x)=\left(\frac{7-y}{2}\right)2$$

$$2x=7-y$$

$$2x+y=7$$

(B) Square both sides and multiply by 3.

$$\left(\sqrt{2t+r}\right)^2=(5)^2$$

$$2t+r=25$$

$$6t+3r=75$$

Distribute:

$(b+7)(b-10)$

Answer: $b^2 - 3b - 70$

Use FOIL—First, Outer, Inner, Last

$(b)(b) + (b)(-10) + (7)(b) + (7)(-10)$

$b^2 - 10b + 7b - 70$

$b^2 - 3b - 70$

If 2 is one solution to the equation $x^2 - 9x + c = 0$, where c is a constant, what is the other solution?

Answer: 7

Work backwards—even though we do not know the value of c, since 2 is one solution, we know the factored form of the quadratic is $(x - 2)(x - ?)$. We also know that the two numbers in parentheses must add to -9. Therefore the factored form is $(x - 2)(x - 7)$ and the other solution is $x = 7$.

This problem can also be solved by plugging $x = 2$ into the original equation and solving for c, then factoring the resulting equation $(x^2 - 9x + 14 = 0)$.

What error has been made?

$$x^2 = 36$$

$$\sqrt{x^2} = \sqrt{36}$$

$$x = 6$$

Answer:

Remember, $\sqrt{x^2} = |x|$. So after we take the square root of both sides, we have $|x| = 6$.

This gives two possibilities: $x = 6$ or $x = -6$.

Alternatively, simply recall that there are generally two possible solutions in exponential equations with an even exponent. Thus when $x^2 = 36$, $x = 6$ or -6.

If $c < 4$, what is the range of possible values of d for the equation $3c = -6d$?

Answer: $d > -2$

If we isolate c in the equation, we can then substitute into the inequality to find the range of d.

$3c = -6d$

$c = -2d$

Now replace (c) with $(-2d)$ in the inequality:

$(-2d) < 4$

$d > -2$

What are the roots of $x^3 - x = 0$?

Answer: $x = 0, -1, \text{or } 1$

Factor the equation, since we already have 0 on one side:

$$x(x^2 - 1) = 0$$
$$x(x + 1)(x - 1) = 0$$
$$x = 0, -1, \text{or } 1.$$

The temptation is to move x to the other side and divide both sides by x, leaving us with $x^2 = 1$. Avoid dividing away a variable unless you *know* it does not equal 0.

Consider the formula $H = \frac{2a^3}{b}$.

If a is doubled and b is increased by a factor of 4, by what factor is H increased?

Answer: *H* is increased by a factor of 2 (*H* is doubled)

The exponent of 3 on a means when we double a, the whole formula gets multiplied by 2^3, or 8. b has no exponent, but it is in the denominator, so quadrupling it is the equivalent of multiplying the formula by 1/4. Thus, H gets multiplied by $8 \times 1/4 = 2$.

What is x? (Hint: Try a method other than substitution)

$$x + y = 10$$

$$3x - 5y = 6$$

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Answer: 7

One way to solve for a variable when you have two equations is to combine the equations in a way that eliminates the *other* variable, (here, y). In this case, we can multiply the first equation by 5, and then add it to the second equation, giving us:

$$\begin{array}{l} 5x + 5y = 50 \\ \underline{3x - 5y = 6} \\ 8x + 0y = 56 \quad \longrightarrow \quad x = 7 \end{array}$$

On the GRE, combination is often faster than substitution.

If x and y are both less than 5, what is the maximum value for the product xy?

Answer: Positive infinity (i.e. there is no maximum value)

Example: $(-10)(-4) = 40$

$(-200)(-32) = 6{,}400$

$(-500{,}000)(-11{,}000) = 5{,}500{,}000{,}000$

Don't forget about negative numbers!

Solve for w:

$$2^{2w} = 8^{w-5}$$

Answer: $w = 15$

We must first obtain the same base on both sides. Convert the 8 into a power of 2:

$2^{2w} = (2^3)^{w-5} \qquad 2^{2w} = 2^{3w-15}$

Now that the bases are equal, we can set the exponents equal to each other:

$2w = 3w - 15 \longrightarrow w = 15.$

Solve:

$$(x-4)^2 = 49$$

Answer: $x = 11$ or -3

Do not multiply out $(x - 4)^2$ if there is a perfect square on one side of the equation. Instead, take the square root of both sides, and remember to place the side of the equation containing the unknown in an absolute value. $|x - 4| = 7$. Our two solutions to this equation are $x - 4 = 7$ and $x - 4 = -7$. Solving these two equations gives us $x = 11$ and -3.

The first few steps of a problem are shown. Finish the problem and answer the question: what is x?

$$\sqrt{x+3} = x - 3$$
$$x + 3 = (x - 3)^2$$
$$x + 3 = x^2 - 6x + 9$$
$$0 = x^2 - 7x + 6$$

Answer: $x = 6$ (x does NOT equal 1!)

Although this equation can be simplified and factored into $(x - 6)(x - 1) = 0$, you need to be careful. When you square an equation containing a variable, you may create extraneous solutions. Potential answers need to be plugged back in to the original equation and verified. 6 is a genuine solution; 1 is not.

Try plugging 1 back into the original equation to verify that x cannot equal 1.

$$\sqrt{1+3} \neq 1-3$$
$$\sqrt{4} \neq -2$$

The square root symbol always indicates the *positive* root.

What is $x + y + z$?

$$x + y = 8$$

$$x + z = 11$$

$$y + z = 7$$

Answer: 13

There is often a faster method than solving for the value of each variable. In this case, we can simply add all the equations together!

$$
\begin{array}{rcl}
x + y \phantom{{}+z} & = & 8 \\
x \phantom{{}+y} + z & = & 11 \\
y + z & = & 7 \\
\hline
2x + 2y + 2z & = & 26 \\
x + y + z & = & 13
\end{array}
$$

Remember, $x + y + z$ is a "combo." In this type of problem there is a good chance you will not need to determine the individual values of the variables.

Simplify:

$$(\sqrt{2} + 3)(\sqrt{2} - 3)(2 - \sqrt{3})(2 + \sqrt{3})$$

Answer: -7

Remember, $(a + b)(a - b) = a^2 - b^2$.

Therefore, our expression is equal to:

$(2 - 9) \times (4 - 3) = (-7)(1) = -7$

A group of rabbits grows by a constant factor every day. If the population grew from 200 to 5,000 in one week, by what factor does its population increase every day?

Answer: $\sqrt[7]{25}$ or $25^{1/7}$

The number of rabbits at the beginning of the day times some factor x equals the new number of rabbits. After a week the original value will have been multiplied by x^7:

$$200(x^7) = 50{,}000$$

$$x^7 = \frac{50{,}000}{2} = 25$$

$$x = \sqrt[7]{25}$$

If $a \neq b$, $a > 0$ and $b > 0$, simplify:

$$\frac{a-b}{\sqrt{a}+\sqrt{b}}$$

Answer: $\sqrt{a}-\sqrt{b}$

When there is a square root term in the denominator that is added to or subtracted from another term we can multiply by the conjugate (the same expression, but with the sign on the 2nd term flipped) to simplify:

$$\frac{a-b}{\sqrt{a}+\sqrt{b}}\left(\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)=\frac{(a-b)\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)}=\frac{(a-b)\left(\sqrt{a}-\sqrt{b}\right)}{(a-b)}=\sqrt{a}-\sqrt{b}$$

Alternatively, you could use the special product $a^2 - b^2 = (a + b)(a - b)$ to solve. In this case, $a-b=\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$, and so the term $\sqrt{a}+\sqrt{b}$ would cancel from the top and bottom, leaving $\sqrt{a}-\sqrt{b}$.

Solve for x:

$$\frac{3x}{3y+5z}=8$$

$$6y+10z=18$$

Answer: $x = 24$

Divide the second equation by 2 and get $3y + 5z = 9$. Substitute 9 for the denominator of the fraction in the first equation. This gives $\frac{3x}{9}=8$, which reduces to $\frac{x}{3}=8$, and finally $x = 24$.

Remember, when you see 3 variables and only 2 equations, you should not automatically assume that you cannot solve for a particular value.

Simplify:

$$\frac{3}{2+\sqrt{3}}$$

Answer: $6-3\sqrt{3}$

To remove a square root from a denominator of the form $a+\sqrt{b}$, multiply the fraction by $\frac{a-\sqrt{b}}{a-\sqrt{b}}$. The form is the same whether you are dealing with numbers, variables, or a combination of the two.

$$\frac{3}{2+\sqrt{3}}\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)=\frac{3\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}=\frac{6-3\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-3}=\frac{6-3\sqrt{3}}{1}=6-3\sqrt{3}.$$

In a sequence of terms, each term after the first is 23 times the previous term. If the 19th term is 40, what is the 11th term?

Answer: $\frac{40}{23^8}$

We could simply work backwards from the 19^{th} term, dividing each term by 23. $S_{18} = 40/23$, $S_{17} = 40/23^2$, etc.

Generally, in a sequence, if you know the factor that each term is being multiplied by 23 in this case, and if you know just one term, it is sufficient to solve for any other term in the sequence.

Look for a pattern. Each term before the 19^{th} is 40 divided by some power of 23. The power of 23 is determined by how many terms prior to the 19^{th} the term is. For example, S_{16} would be $\frac{40}{23^3}$ because it is 19 – 16 = 3 terms prior to 19. Since S_{11} is 19 – 11 = 8 terms prior to 19, $S_{11} = \frac{40}{23^8}$.

How would you factor each of the following expressions?

(A) $x^5 - x^3$

(B) $4^8 + 4^9 + 4^{10}$

(C) $m^{n-2} - 3m^n + 4m^{n+1}$

Answer:

(A) The GCF is x^3, the smaller power.

$x^3(x^2 - 1) = x^3\ (x + 1)(x - 1)$.

(B) The GCF is 4^8. $4^8(1 + 4^1 + 4^2) = 4^8(21)$.

(C) The smallest power of m is the GCF. Here it is m^{n-2}:

$m^{n-2}(1 - 3m^2 + 4m^3)$.

Solve by picking numbers:

Bottle 1, with capacity x liters, is half full of water. Bottle 2, with capacity y liters, is one sixth full. If Bottle 2 is three times the size of Bottle 1 and the contents of Bottle 1 are emptied into Bottle 2, how many liters of water, in terms of y, are in Bottle 2?

(A) $\frac{1}{2}y$ (B) $\frac{1}{6}y$ (C) $\frac{2}{3}y$ (D) $\frac{1}{3}y$ (E) $\frac{5}{6}y$

Answer: (D) $\frac{1}{3}y$

When problems involve many fractions and no specific quantities, pick numbers that are multiples of all the denominators in the problem. The least common multiple of 6 and 2 is 6. Thus, let the capacity of Bottle 1 = 6 and the capacity of Bottle 2 = 18. Bottle 1 holds 3 liters and bottle 2 holds 3 liters. Bottle 1 is dumped into Bottle 2, which then contains 6 liters. Test each answer choice with $y = 18$ and notice that (D) is the solution, since $\frac{1}{3}(18) = 6$.

Set up an appropriate equation to describe the given scenario:

The elasticity (e) of a material is directly proportional to the square of its density (d) and inversely proportional to the cube of its mass (m).

Answer: $e=\frac{kd^2}{m^3}$

A constant *k* is used in expressions of direct or inverse proportionality. *e* is directly proportional to d^2, which means $e = kd^2$. *e* is also inversely proportional to m^3, so $e = k/m^3$. Putting these two equations together, we get $e=\frac{kd^2}{m^3}$.

Note that *k* in the final equation must be the product of the *k* constants in the first two equations, but since *k* could be any value, we can repeat the use of *k* for simplicity.

If $x + y = 5$ and $x^2 - y^2 = 20$, what is y?

Answer: $y = 0.5$

First, factor $x^2 - y^2$:

$(x + y)(x - y) = 20$

Since $(x + y) = 5$, $(x - y) = 4$.

Add these two linear equations, to cancel y: $2x = 9$, so $x = 4.5$

Plugging into $x + y = 5$, if $x = 4.5$, then $y = 0.5$.

Identify the error:

$8!+2 \leq x \leq 8!+10$ implies that $2 \leq x \leq 10$.

Answer: $2 \le x - 8! \le 10$ is the correct result.

In a compound inequality, you must perform the same operation to *all 3 expressions*, not just the outside expressions. If you subtract 8! from all 3 expressions, you get $2 \le x - 8! \le 10$.

If $xz > 0$ and $yz < 0$, what can you conclude about xy?

Answer: $xy < 0$

$xz > 0$ means x and z have the same sign. $yz < 0$ means y and z have opposite signs. Together, this means that x and y must have opposite signs and consequently $xy < 0$.

Factor:

$$\frac{x^2}{9} - 25y^2$$

Answer: $\left(\frac{x}{3}+5y\right)\left(\frac{x}{3}-5y\right)$

This expression is a slightly more complicated version of the special product $a^2 - b^2 = (a + b)(a - b)$. Notice that x^2, 9, 25, and y^2 are all perfect squares:

$$\frac{x^2}{9}-25y^2=\left(\frac{x}{3}\right)^2-(5y)^2=\left(\frac{x}{3}+5y\right)\left(\frac{x}{3}-5y\right)$$

The first three terms of a linear sequence are −2, 18, and 38. What is the rule for this sequence?

Answer: $S_n = 20n - 22$, for integer $n > 0$

Since the terms are increasing by 20, we know the rule is $S_n = 20n + k$. Use any of the three given terms to solve for k:

$S_2 = 20(2) + k \quad 18 = 40 + k \quad k = -22$

The rule is $S_n = 20n - 22$.

Try solving for k using S_1 or S_3 and verify that you get the same value. Could you express this sequence using a recursive definition?

If $10 \leq m \leq 20$ and $-2 \leq p \leq 15$ and m and p are both integers, what is the maximum possible value for $m - p$?

Answer: 22

To maximize $m - p$, make m as large as possible and make p as small as possible. $m = 20$ and $p = -2$.

$20 - (-2) = 22$.

Solve by picking numbers for x and y, then testing the answer choices.

What is the average of $(x + y)^2$ and $(x - y)^2$?

(A) $2x^2 - 2y^2$

(B) $x^2 + 4xy + y^2$

(C) $x^2 + y^2$

Answer: (C) $x^2 + y^2$

Let's pick $x = 3$ and $y = 2$.

$(3 + 2)^2 = 25$ and $(3 - 2)^2 = 1$, so the average is $\frac{25+1}{2} = 13$.

Now, let's test each answer choice:

A) $2(3)^2 - 2(2)^2 = 10$

B) $(3)^2 + 4(3)(2) + (2)^2 = 37$

C) $(3)^2 + (2)^2 = 13$

Alternatively, expand the expressions:

$(x + y)^2 = x^2 + 2xy + y^2$

Everyone in a certain office orders a cup of coffee. The ratio of cappuccinos to lattes to espressos ordered is 1 : 2 : 3. If there are 60 people in the office, how many cups of each type of coffee were ordered?

Answer: 10 cappuccinos, 20 lattes, 30 espressos

Using the unknown multiplier, we can set up the equation $1x + 2x + 3x = 60$, since 60 cups of coffee were ordered (one for each person). Solving for x, we find that $x = 10$, and then we can apply that multiple to each element in the proportion.

The average of 177, 176, 189 and x is 180. What is x?

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Answer: 178

A shortcut to dealing with averages is to focus on how much above or below the average every number in the set is. Then, we can "balance" this difference in the final term. In this example, 177 is 3 below the average, 176 is 4 below the average, and 189 is 9 above the average.

$-3 - 4 + 9 = +2$, so the fourth number must be 2 *below* the average to balance it out. 178 is 2 below 180.

This method is *much* easier than applying the definition of averages:

$\frac{177+176+189+x}{4}=180$. Ugh.

Which is the correct expression?

The phone call pricing for a long distance company is as follows: \$5.00 for the first minute, and \$0.15 for every additional minute. After 10 minutes, the price drops to \$0.10 per minute. How much does a 17 minute phone call cost, in dollars?

(A) $5 + 10(0.15) + 7(0.10)$

(B) $5(10) + 0.15 + 7(0.10)$

(C) $5 + 9(0.15) + 7(0.10)$

Answer: (C) $5 + 9(0.15) + 7(0.10)$

We know that the first minute costs $5.00. The next 9 minutes (not 10—don't forget to subtract out the first minute!) will be charged at the rate of $0.15 per minute. After that, the next 7 minutes will be charged at the rate of $0.10 per minute.

In a round of miniature golf, the probability that Jasper will get a hole in one is 1/13. The probability that Cornelius will get a hole in one is 1/12. If these probabilities are independent, what is the probability that neither of them will get a hole in one on the next hole?

Answer: 11/13

If Jasper has a 1/13 chance of getting a hole in one, that means he has a 12/13 chance of not getting it. Similarly, Cornelius has an 11/12 chance of not getting the hole in one. Because we want the probability of Jasper not getting the hole in one AND Cornelius not getting the hole in one, we multiply the probabilities:

$$\frac{12}{13} \times \frac{11}{12} = \frac{11}{13}.$$

This equation only holds if the two occurrences are independent (like successive coin flips), but many, if not most, probability problems involving more than one event assume independence.

Which of the following changes to a set of at least 3 consecutive positive integers will result in a list whose median and mean are different?

(A) Every number in the list is tripled

(B) Every number in the list has 12 added to it

(C) Every number in the list is squared

Answer: (C) Every number in the list is squared

As long as a set of numbers is evenly spaced, its average will equal its median. The changes described in answer choices (A) and (B) would keep the numbers in the lists equally spaced. Only answer choice (C) would change the spacing.

1, 2, 3 $\longrightarrow$ 3, 6, 9 evenly spaced

1, 2, 3 $\longrightarrow$ 13, 14, 15 evenly spaced

1, 2, 3 $\longrightarrow$ 1, 4, 9 not evenly spaced

Initially, the ratio of potbellied pigs to carrots in a room had been 1 : 60. After the potbellied pigs ate most of the carrots, though, the new ratio was 3 : 1. If there were 6 potbellied pigs in the room, how many carrots did they eat, total?

Answer: 358 carrots

From the first ratio, we know that there were originally 360 carrots, since 1/60 = 6/360. The second ratio tells us that the potbellied pigs only left 2 carrots uneaten, since 3/1 = 6/2. We can calculate that 360 carrots – 2 uneaten carrots = 358 carrots.

Sean is 15 years older than Eric. In 6 years Sean will be twice as old as Eric. How old is Eric?

(A) 9
(B) 14
(C) 24

Answer: (A) 9

An alternative approach to setting up equations and solving for the appropriate variable is to use the answer choices to help you. To demonstrate using the right answer, we begin by assuming that Eric is 9. From the first sentence, we know that Sean is 24. In 6 years, Eric will be 15 and Sean will be 30. 30 is indeed 2 times 15, so we know that 9 is the right answer.

For this type of question, using the answer choices is an alternate approach to setting up the equations. To do very well on the GRE, it's important to know how to setup and solve the algebra, but you should also have other tools available to you.

Give an example of a set of numbers in which 75% of the numbers are greater than or equal to the median.

Answer: Can vary, but any set like {1, 2 | 3, 3 | 3, 3 | 5, 9} or {50 | 51 | 51 | 109} {1, 2, 3 | 5, 5, 5 | 5, 6, 7 | 8, 9, 10}. The | mark separates quartiles.

In order for 75% of the terms to be a whole number of terms, the number of terms in the set must be a multiple of 4. So, the number of terms in the set must be even, and the median is the average of the two middle terms.

A key phrase is "...or equal to the median." By definition, the median is greater than or equal to 50% of the terms, and less than or equal to 50% of the terms. In order for 75% of the terms to be greater than or equal to the median, the terms in the 2nd quartile must equal the median.

Jeff completed a 40 mile circuit around the city, jogging at a constant speed of 5 mph. Then Jeff completed the same circuit again, but this time he biked at a constant speed of 20 mph. What was Jeff's average speed for the total trip?

Answer: 8 mph

To calculate average speed, you need the total distance traveled and the total time spent traveling. The distance is straightforward. Jeff traveled 40 miles twice, for a total distance of 80 miles. If we divide distance by speed, we find that it took him $40/5 = 8$ hours to jog the circuit, while it took him $40/20 = 2$ hours on the bike, for a total time of 10 hours.

Finally, the average speed is the total distance divided by the total time:

$$\frac{80\ \text{miles}}{10\ \text{hours}} = 8\text{mph}.$$

Greg and Liz are currently 350 miles apart. They begin driving toward each other, Greg driving 60 mph and Liz driving 40 mph. How long will it take until they meet?

Answer: 3.5 hours

When dealing with two objects moving toward each other, we can simplify the calculations by adding the rates together. If Greg is moving 60 mph and Liz is moving 40 mph, then together they are traveling 100 miles per hour relative to one another. At that speed, they will collectively travel 350 miles in 3.5 hours, because T = (350 miles)/(100 mph) = 3.5.

Which is the correct expression for this problem?

8 students have been chosen to play for PCU's inter-collegiate basketball team. If every person on the team has an equal chance of starting, what is the probability that both Tom and Alex will start? (Assume 5 starting positions)

(A) $\dfrac{\frac{6!}{3!3!}}{\frac{8!}{5!3!}}$ (B) $\dfrac{6\times5\times4}{8\times7\times6\times5\times4}$ (C) $\dfrac{\frac{6!}{3!3!}}{8\times7\times6\times5\times4}$

Answer: (A) $\dfrac{\frac{6!}{3!3!}}{\frac{8!}{5!3!}}$

First, count all the combinations of 5 people starting, given 8 people on the team. The formula is $\frac{\text{Team}!}{\text{Start}!\,\text{Not}!} = \frac{8!}{5!3!}$. Next, to count the combinations that include Tom and Alex, assume that these 2 players start, then choose the remaining 3 starters from among the 6 remaining people. The formula is the same but takes different numbers: $\frac{\text{Team}!}{\text{Start}!\,\text{Not}!} = \frac{6!}{3!3!}$. Finally, divide the second count by the first count, since Probability $= \frac{\text{Successful Combinations}}{\text{All Combinations}}$ (as long as each combination is equally possible).

5 people in a company earn an average salary of \$50,000. If 2 of the employees earn an average of \$65,000, what is the average salary of the remaining 3 employees?

Answer: $40,000

One approach to this problem is to balance the "overs" and the "unders." The 2 employees making $65,000 each make $15,000 more than the mean, for a total of $30,000 over the mean. That means the remaining 3 employees need to make a combined $30,000 under the mean. Distribute that amount evenly, and the remaining salaries average out to $10,000 below the mean, or $40,000.

10 years ago Tina was half as old as Ron will be 7 years from now. Which equation correctly represents this relationship?

(A) $(1/2)(T - 10) = (R + 7)$

(B) $(T - 10) = (1/2)(R + 7)$

Answer: (B) $(T - 10) = (1/2)(R + 7)$

When you translate sentences into equations, one of the easiest mistakes to make is to put the multiplier in the wrong place. Take the time to verify that it is where it should be. In this case, if Tina's age 10 years ago was half what Ron's age will be in 7 years, we need to multiply Ron's age by 1/2 to make them equal.

Don't forget to use the parentheses!

Jerry and Ross decide to have a footrace. They run 1,000 meters. Jerry runs 5 meters per second, and Ross runs 4 meters per second. Halfway through the race, Jerry realizes he is ahead and stops running for one full minute before finishing the race at his original speed. Who wins the race?

Answer: Ross wins

First, calculate how long it will take Ross to finish the race. $T = D/R$. T = (1,000 m)/(4m/s) = 250 s. To make the calculations simpler for Jerry, add 60 seconds to the total time to take into account the minute he spent not running. T = (1,000 m)/(5 m/s) + 60 s = 260 s. Ross finishes the race in less time.

What is the average of 3,456, 3,463, 3,470, 3,477, and 3,484?

Answer: 3,470

When a list is composed of evenly spaced numbers, the average will equal the median. In this case, all the numbers in the list are spaced 7 units apart. The median is 3,470, and the average is also 3,470.

The original ratio of girls to boys in a class was 2 : 3. Then 6 girls were added to the class, bringing the ratio of girls to boys in the class to 1 : 1. How many students are now in the class?

Answer: 36

Begin with the unknown multiplier. If the original ratio of boys to girls is 2 : 3, then the original number of girls is $2x$ and the original number of boys is $3x$. After 6 girls are added, the new ratio is 1:1, so we can write the equation $\frac{2x+6}{3x}=\frac{1}{1}$. Cross-multiply to get $2x + 6 = 3x$, which simplifies to $x = 6$. So the original number of girls was $2x = 2(6) = 12$ and the original number of boys was $3x = 3(6) = 18$. There were $12 + 18 = 30$ students in the class before the 6 girls were added. There are now $30 + 6 = 36$ students in the class.

Car A travels east at 60 mph. Car B is 45 miles behind Car A and also travels east at 75 mph. How long will it take Car B to catch up with Car A?

Answer: 3 hours

When solving a combined rates problem with 2 objects moving in the same direction, we can ignore their actual speeds and focus only on the difference between their speeds. Car B is going 15 mph faster than Car A. That means it will catch up to Car A at a rate of 15 mph. Car B is currently 45 miles behind Car A, and $R \times T = D$, so

(15 mph) $\times$ T = 45 miles. Solving for T gives us T = 3 hours.

A shipping company charges $5 + 10/y^2$ dollars per package shipped by a customer over a given month, where y is the number of packages shipped that month by the customer. If a customer spends \$51 one month on shipping, which equation will correctly solve for the number of packages shipped that month by the customer?

(A) $5y + 10 = 51$

(B) $5 + 10/y = 51$

(C) $5y + 10/y = 51$

Answer: (C) $5y + 10/y = 51$

We know that y represents the number of packages shipped by the customer in the month, and $5 + 10/y^2$ is the shipping cost per package for that customer. Therefore, the product of these two terms must equal 51:

$$(y)(5 + 10/y^2) = 51$$
$$5y + 10/y = 51$$

For added practice, what is the value of y? Try choosing integers that make sense.

Jeff can build a doghouse in 6 hours. Kevin can build the same doghouse in 3 hours. How long will it take them, working together, to build 1 doghouse?

Answer: 2 hours

Whenever you are told how long it takes a person to complete a task, a great first step is to turn that information into a rate. In an $R \times T = W$ equation, we can think of completing the task as doing 1 unit of work. If it takes Jeff 6 hours to build the doghouse, then his rate is 1/6 of the doghouse per hour. Similarly, Kevin's rate of work is 1/3 of the doghouse per hour. When we combine their rates (because they're working together), we see that they complete 1/2 of the doghouse every hour, because $1/6 + 1/3 = 1/2$. At that rate, they will complete the doghouse in 2 hours.

In the World's Strongest Man competition, Olav Gundersson managed to pull a truck a total distance of 85 ft combined in two tries. On his second try, he pulled the truck 10 ft more than half the distance he pulled the truck on his first try. How far did he pull the truck on his first try?

(A) 45 ft (B) 50 ft (C) 55 ft (D) 60 ft (E) 65 ft

Answer: (B) 50 ft

We can set this problem up and solve algebraically. We know that the sum of the two tries equals 85 feet, and the second try is 10 more than half the first try. Let's use x to represent the first try and y to represent the second try:

$x + (10 + 0.5x) = 85$

$y = 10 + 0.5x$

$x + (10 + 0.5x) = 85$

$1.5x = 75$

$x = 50$

Alternatively, plug the answer choices and eliminate.

John and Fawn, each drinking at a constant pace, can finish a case of soda together in 12 hours. If each were drinking the soda alone, it would take John 10 hours longer to finish the case of soda than it would take Fawn. In how many hours can John finish a case of soda, drinking alone?

(A) 24 hours (B) 30 hours (C) 36 hours (D) 42 hours

Answer: 30 hours

For all work problems, $R = \frac{W}{T}$. Here, the "work" is drinking one case of soda, so W = 1.

This problem provides three equations about rates: $\left(J + F = \frac{1}{12}\right)$ and $J = \frac{1}{t}$ and $\left(F = \frac{1}{t-10}\right)$ where J is John's rate, F is Fawn's rate, and t is John's time.

To avoid ugly algebra, work backwards from answer choices. Say we start with (B). John's time $t = 30$, so his rate J is 1/30. Fawn's time is $t - 10 = 20$, so her rate F is 1/20. Now check whether 1/30 + 1/20 = 1/12. This is true (1/30 + 1/20 = 2/60 + 3/60 = 5/60 = 1/12), so (B) is the answer.

A bag contains ping pong balls, each with a number written on it. The average of all the numbers is 50. When $\frac{1}{3}$ of the ping pong balls are removed, the average of the numbers written on the removed balls is 20. What is the average of the numbers on the balls still remaining in the bag?

Answer: 65

Although we don't know how many ping pong balls are in the bag, we can modify the weighted averages formula to help us answer this question. The weighted average equals the sum of the weights times the data points, divided by the sum of the weights. So, $\frac{20}{3}+\frac{2}{3}x=\frac{150}{3}$, or $\frac{2}{3}x=\frac{130}{3}$, and finally $x=\frac{130}{3}\times\frac{3}{2}=\frac{130}{2}=65$.

Alternatively, imagine that we start with 3 balls. Remove 1/3 of 3, or 1 ball, which has a 20 on it. The average is 50, so the sum is 3 × 50 = 150. The other two balls must add to 150 – 20 = 130. 130 averaged over 2 balls is 65.

At a farmer's market, one stall is selling mangos, kumquats and rutabagas. The ratio of mangos to kumquats is 5 : 4, and the ratio of kumquats to rutabagas is 3 : 7. Which of the following could be the number of kumquats?

(A) 28 (B) 36 (C) 42 (D) 49

Answer: (B) 36

A common hidden constraint of word problems involving ratios is that the actual number of items must be an integer. For instance, it would not make sense in this context to have 5 1/3 mangos. The correct answer will be a multiple of both 4 and 3, based on the ratios involving kumquats.

For added practice, if there are actually 36 kumquats, how many mangos and rutabagas are there?

Train A and Train B start from 500 miles apart and travel towards each other on identical parallel tracks at constant speeds. They meet in 5 hours, but if Train B were going twice as fast as it actually is, the two trains would meet in 4 hours. At what rate is Train A actually traveling?

Answer: 75 mph

First, label the speed of Train A as R_A and the speed of Train B as R_B. When dealing with objects moving towards each other, we can add their rates, so $(R_A + R_B) \times (5 \text{ hrs}) = 500$ miles. The problem also states that $(R_A + 2R_B) \times (4 \text{ hrs}) = 500$ miles.

Dividing by the times:

$R_A + R_B = 100$ mph

$R_A + 2R_B = 125$ mph

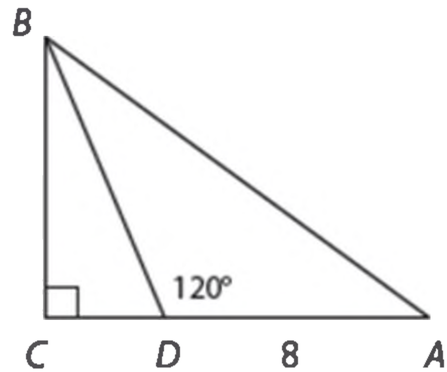
Subtracting the first equation from the second:

$R_B = 25$ mph

Therefore, $R_A = 75$ mph.

What is the perimeter of Triangle *BCD*?

$\angle ABD = \angle DAB$



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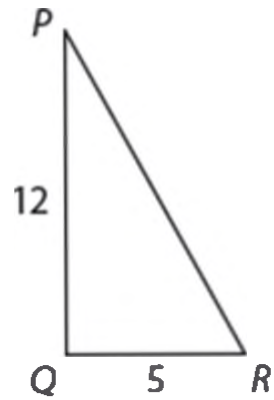
Answer: $12+4\sqrt{3}$

Because $\angle ABD = \angle DAB$, side *BD* has length 8. Because $\angle BDA$ is 120°, $\angle BDC$ is 60°, so triangle *BCD* is a 30–60–90 triangle.

Side	Ratio	Length
CD	x	4
BC	$x\sqrt{3}$	$4\sqrt{3}$
BD	$2x$	8

Sum the sides to find the perimeter: $8 + 4 + 4\sqrt{3} = 12+4\sqrt{3}$

If the area of triangle PQR is 30, and PR is the longest side of the triangle, what is the measure of $\angle PQR$?



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Answer: 90°

Notice that $\frac{1}{2}(5)(12) = 30$. QR is the base of the triangle and PQ is the height. Base and height of a triangle *must* be perpendicular to each other. Therefore $\angle PQR$ is a right angle.

If PQR were anything other than a right triangle, the height of the triangle would be less than 12, and the area would be less than 30.

What is the area of a circle that has a circumference of 14π?

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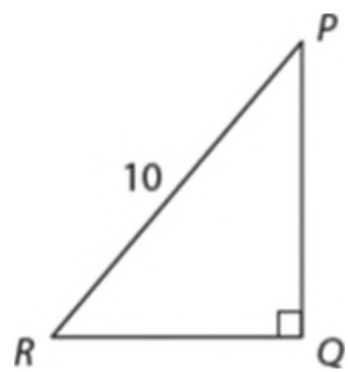
Answer: 49π

The key to this problem is to find the radius, because the radius is used to calculate both circumference and area.

$C = 2\pi r$, so $14\pi = 2\pi r$. Therefore $r = 7$.
$A = \pi r^2$, so $A = \pi(7)^2 = 49\pi$

The ratio of side *PR* to side *PQ* is 5 : 4. What is the length of side *QR*?



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Answer: 6

First, complete the ratio. 5/4 = 10/*PQ*. Side *PQ* has length 8. With two sides of a right triangle, you can use the Pythagorean theorem to find the third side. In this case, however, you should recognize that this triangle is a common Pythagorean triplet—a 6–8–10 triangle. Side *QR* has length 6.

A right circular cylinder has a volume of 10π. If the radius is doubled and the height remains the same, what is the volume of the new cylinder?

Answer: 40π

$V = \pi r^2 h$, so $10\pi = \pi r^2 h$. We are told the radius is doubled, so the new radius is $2r$. Replace r with $2r$ and the new equation is $V = \pi(2r)^2 h$. The new volume is $V = 4\pi r^2 h$. Since we know that $\pi r^2 h = 10\pi$, we know that $V = 4(10\pi) = 40\pi$.

Alternately, try picking numbers for the cylinder that would produce an initial volume of 10π; a height of 10 and a radius of 1 are simplest. Double the radius and calculate the new volume.

What is the surface area of a cube with side length 5?

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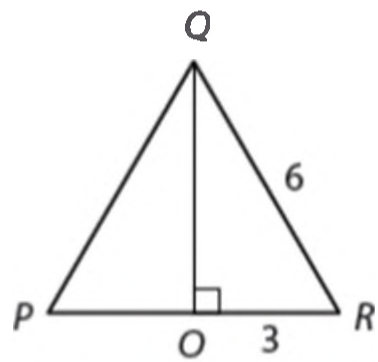
Answer: 150

Surface area is the sum of all the areas of the faces of the cube. The faces of a cube are all identical, so the surface area is the area of one of the faces times 6, because a cube has 6 faces. The area of one face is A = length × width.

Therefore, surface area = $6 \times 5 \times 5 = 150$.

If $PQ = QR$, what is the area of triangle PQR?



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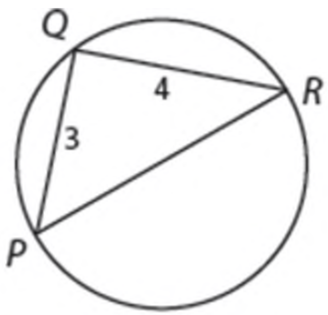
Answer: $9\sqrt{3}$

Triangle *OQR* is a right triangle, and we have 2 sides, so either by using the Pythagorean Theorem or by recognizing it as a 30–60–90 triangle, we can find that the length of *OQ* is $3\sqrt{3}$, which is also the height of triangle *PQR*. If *PQ* = *QR*, then ∠*OPQ* = ∠*ORQ*. Additionally, we know that ∠*POQ* = ∠*ROQ* = 90°. Therefore ∠*OQP* = ∠*OQR*, because when two triangles have two angles in common, they must have common third angles. Triangles *OQR* and *OQP* are thus similar triangles, and also have identical side lengths. *OP* must equal 3, so we have the length of the base as well as the height. The area of triangle *PQR* is $\frac{1}{2}bh=\frac{1}{2}(3+3)(3\sqrt{3})=9\sqrt{3}$.

If $\angle PQR$ is a right angle, what is the area of the circle?



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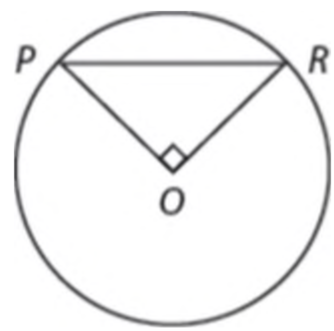
Answer: $\frac{25}{4}\pi$ or 6.25π

If $\angle PQR$ is a right angle, then segment PR must be a diameter of the circle. We can solve for the length of PR using the Pythagorean Theorem, or by recognizing that triangle PQR is a 3–4–5 triangle. If the diameter of the circle is 5, then the radius is 2.5. The area of the circle is $A = \pi r^2 = \pi\left(\frac{5}{2}\right)^2 = \frac{25}{4}\pi = 6.25\pi$

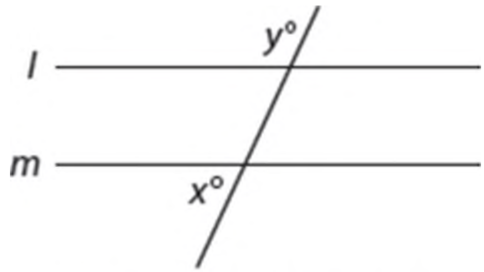
O is the center of the circle. The area of the circle is 81π. What is the length of line segment PR?



Answer: $9\sqrt{2}$

If the area of the circle is 81π, then $81\pi = \pi r^2$, so $r = 9$. *OP* and *OR* are both radii, and so both have length 9. If triangle *OPR* is a right triangle and has 2 sides with equal length, then it is a 45–45–90 triangle, and the ratio of the sides to the hypotenuse is $1 : \sqrt{2}$. Therefore, the length of *PR* is $9\sqrt{2}$.

If $x = 70$, what is y?



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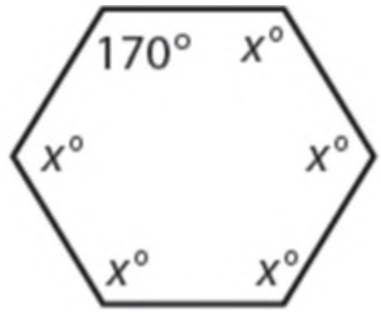
Answer: Maybe 110...maybe not!

Don't trust the picture! Although lines *l* and *m* appear to be parallel, nothing in the question tells us that they are. Unless explicitly told that the lines are parallel, there is no way to determine the value of *y*.

What is x?



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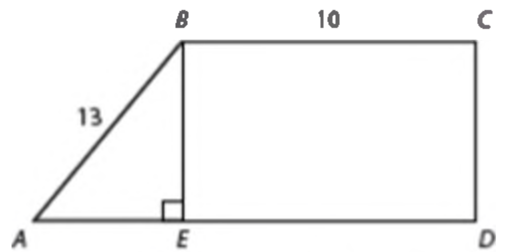
Answer: 110°

The sum of the interior angles of a polygon = $(n - 2) \times 180$, where n is the number of sides.

$(6 - 2) \times 180 = 720$. Therefore, $5x + 170 = 720$.

$5x = 550$, so $x = 110$.

Segment *AD* has length 15. What is the area of rectangle *BCDE*?



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Answer: 120

If $BCDE$ is a rectangle, then $BC = ED$. ED must have a length of 10, so AE has a length of 5. We can use the Pythagorean Theorem to determine the length of BE, or recognize that triangle ABE is a 5–12–13 triangle. Segment BE has a length of 12. Area of a rectangle is $A = \text{base} \times \text{height} = (10) \times (12) = 120$.

An empty right circular cylindrical swimming pool has a height of 10 ft; its base has an area of 15 ft^2. If water fills the pool at a rate of 25 ft^3 every 10 minutes, how long will it take for the pool to be filled?

Answer: 60 Minutes

$V = \pi r^2 h$. Therefore, $V = (15 \text{ ft}^2)(10 \text{ ft}) = 150 \text{ ft}^3$. The pool fills at a rate of 25 ft^3 every 10 minutes, and since $25 \times 6 = 150$, we can proportionally apply this to the rate: $10 \times 6 = 60$ minutes.

What is the length of the main diagonal of a rectangular solid with sides of length 4, 4 and 2?

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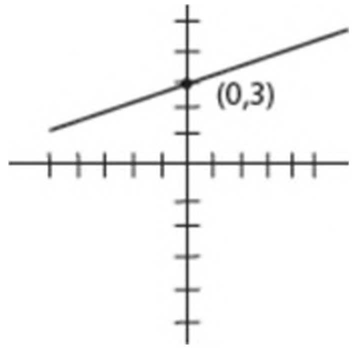
Answer: 6

We could use the Pythagorean Theorem twice, or we can save some time by using the “Deluxe” Pythagorean Theorem to find the interior diagonal of a rectangular prism (a box): $a^2 = x^2 + y^2 + z^2$.

So $d^2 = 4^2 + 4^2 + 2^2 = 16 + 16 + 4 = 36$, and $d = 6$.

If line *l* also passes through point (15, 6), what is the slope of line *l*?



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Answer: $\frac{1}{5}$

Although we cannot easily see the point (15, 6) on the grid given in this problem, we know that two points define a line. Because line *l* goes through point (0, 3) and point (15, 6), we can calculate the slope. The slope is

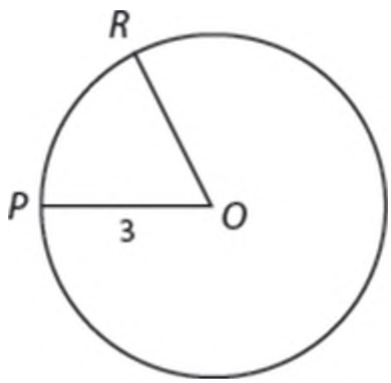
$$\frac{rise}{run}=\frac{\text{change in } y}{\text{change in } x}=\frac{6-3}{15-0}=\frac{3}{15}=\frac{1}{5}$$

The diagonal of a rectangular flowerbed is $15\sqrt{2}$ feet. What is the area of the flowerbed, if its length and width are equal?

Answer: 225 ft^2

If the length and width are equal, then the flowerbed is a square, and the diagonal is the hypotenuse of a 45–45–90 triangle. The ratio of the sides to the hypotenuse is $1 : \sqrt{2}$. If the hypotenuse is $15\sqrt{2}$, then the sides each have length 15, so the area is $l \times w = (15) \times (15) = 225$.

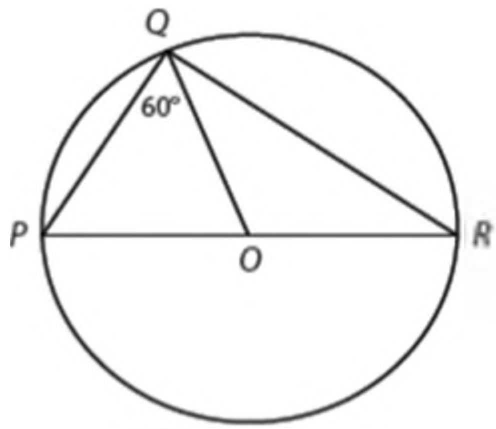
O is the center of the circle. If minor arc PR has a length of π, what is $\angle POR$?



Answer: 60°

To determine $\angle POR$, we first need to determine what fraction of the circumference minor arc PR is. Circumference is $C = 2\pi r$, so $C = 2\pi(3) = 6\pi$. $\pi/6\pi = 1/6$, so minor arc PR is 1/6 of the circumference. That means $\angle POR$ is 1/6 of 360°. $\angle POR$ is 60°.

O is the center of the circle. What is $\angle PRQ$?



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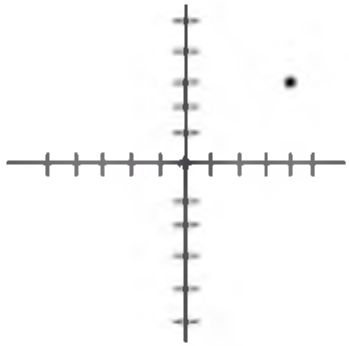
Answer: 30°

First, we can calculate $\angle OPQ$: since OP and OQ are the radii of the circle, they must have equal length, which means the opposite angles in triangle PQO must be equal. Therefore $\angle OPQ = \angle PQO = 60°$.

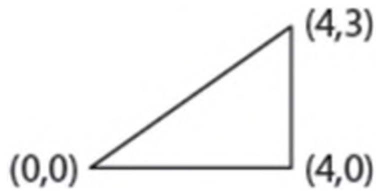
Since PR is a diameter of the circle, triangle PQR must be a right triangle. That means $\angle PQR$ is a right angle, and $\angle OQR$ is $90° - 60° = 30°$.

Lastly, the three angles of triangle PRQ must add up to 180. $\angle PRQ = 180° - 90° - 60° = 30°$.

What is the distance between points (0,0) and (4,3)?

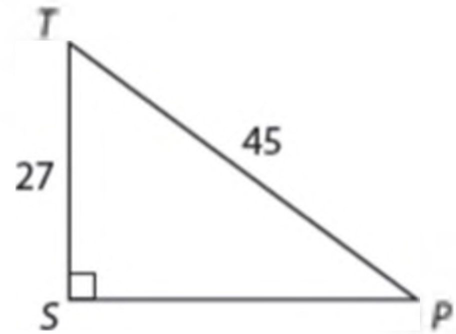


Answer: 5



One way to think of the distance formula is that it's the square root of the Pythagorean Theorem. The hypotenuse is the distance, the difference in the *x*-coordinates is the horizontal leg of the triangle, and the difference in the *y*-coordinates is the vertical leg. In this case, the two legs of the triangle have lengths 4 and 3, respectively. This is a 3–4–5 triangle in disguise. The hypotenuse of the triangle is 5.

What is the length of side *SP*?



Note: Figure not drawn to scale

Answer: 36

Although we can use the Pythagorean theorem, the numbers are large, and the calculation will be time-consuming. Remember that multiples of Pythagorean triplets are also Pythagorean triplets. This is a 3–4–5 triangle in disguise. Every value has been multiplied by 9. $3 \times 9 = 27$ and $5 \times 9 = 45$. The value we're missing is 4×9, which equals 36.

Look for common factors in side lengths such as these, to see whether the sides of the triangle are just multiples of a simpler Pythagorean triplet.

Which of these lines is perpendicular to the line whose equation is $y = \frac{2}{3}x + 6$?

(A) $y = \frac{3}{2}x - 6$

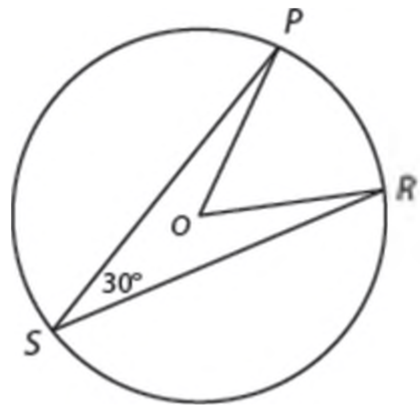
(B) $y = -\frac{2}{3}x + 2$

(C) $y = -\frac{3}{2}x + 7$

Answer: (C) $y = -\frac{3}{2}x + 7$

When determining what lines are perpendicular, there is only one important piece of information—the slope. The slopes of perpendicular lines are negative reciprocals. The slope of the original line is 2/3, so the slope of a perpendicular line will be –3/2. The slope of the line in answer choice (C) is –3/2.

If $\angle POR = 58°$, is point O the center of the circle?



Note: Figure not drawn to scale

Answer: No

$\angle PSR$ is an inscribed angle. If O were the center of the circle, $\angle POR$ would be a central angle and angle $\angle POR$ would be twice $\angle PSR$. Because $58°$ is not twice $30°$, we know that point O cannot be the center of the circle.

Two sides of a triangle have lengths 3 and 9. Which of the following could be the length of the third side?

(A) 6   (B) 8   (C) 13

Answer: (B) 8

In any given triangle, the lengths of any two sides must add up to greater than the third side. Choice(A) fails, because $3 + 6$ is not greater than 9. Choice (C) fails because $3 + 9$ is not greater than 13.

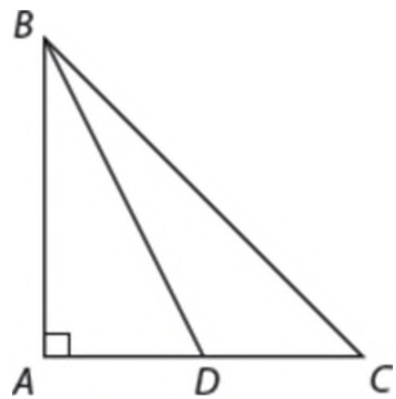
Choice (B) works because:

$3 + 6 > 8$

$3 + 8 > 6$

$6 + 8 > 3$

In triangle ABC, $AB = 12$, $AB = AC$, and BD bisects AC. What is the area of triangle BDC?



Answer: 36

We know that AC has a length of 12, because it is equal to AB. We know that DC has a length of 6, because BD bisects AC. If DC is the base, then AB is the height of triangle BDC. $A = \frac{1}{2}(\text{base}) \times (\text{height})$, so $A = \frac{1}{2}(6) \times (12) = 36$.

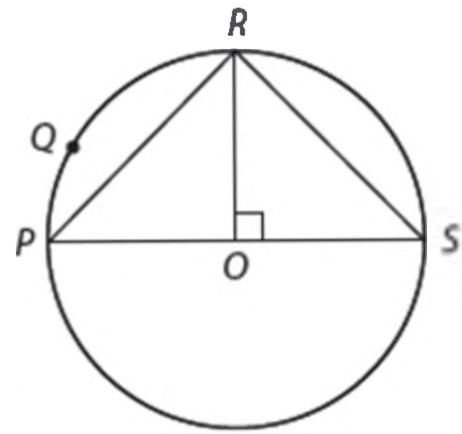
The circumference of the smaller circle is 6π and the circumference of the larger circle is 16π. What is the area of the shaded region?

Note: Figure not drawn to scale.

Answer: 55π

To find the area of the shaded region, we need to find the area of the entire figure and subtract the area of the smaller circle. The area of the entire figure is the area of the larger circle. If the larger circle has a circumference of 16π, then its radius is 8, because $C = 2\pi r$. The formula for area is $A = \pi r^2$, so $A = \pi(8)^2 = 64\pi$. Doing the same calculations for the smaller circle, we find it has a radius of 3 and an area of 9π. $64\pi - 9\pi = 55\pi$.

O is the center of the circle and PS is a diameter of the circle. If the length of arc PQR is 4π, what is the length of line segment RS?

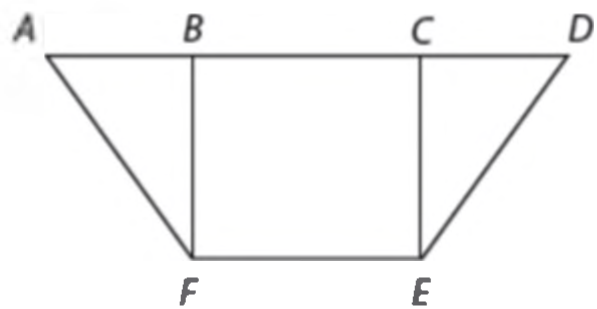


Answer: $8\sqrt{2}$

Since $\angle ROS$ is a right angle, $\angle POR$ is $180 - 90 = 90$ degrees also, and $90°$ is $\frac{1}{4}$ of the circle. This implies that arc $\overset{\frown}{PQR}$ is also $\frac{1}{4}$ of the circle. That means the circumference of the circle is $4 \times 4\pi = 16\pi$. Since $C = 2\pi r = 16\pi$, the radius r is 8.

In triangle ORS, both OR and OS are radii, so they form a right triangle with both legs = 8. ORS is a 45–45–90 triangle, in which the hypotenuse is $\sqrt{2}$ times the leg length. Thus, line segment RS is $8\sqrt{2}$ in length.

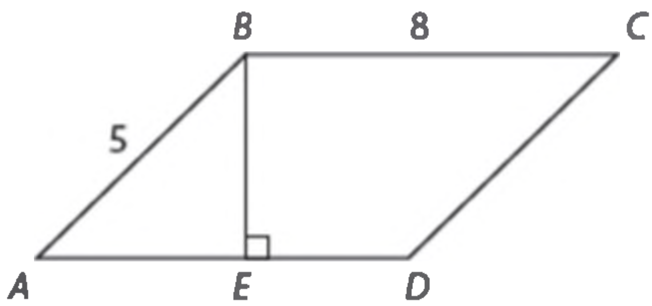
If the area of square *BCEF* is 16 and the length of *AD* is 10, what is the area of trapezoid *ADEF*?



Answer: 28

The area of a trapezoid is $\left(\frac{b_1+b_2}{2}\right)$ × (height), where b_1 and b_2 are the parallel bases of the trapezoid. We can determine the height, because a square that has an area of 16 must have sides of length 4, and BF is the height of the trapezoid. From the statement, we know the average of the lengths of the parallel bases is $\frac{10+4}{2}=7$, so the area is $A = 7 \times 4 = 28$.

Segment *BE* bisects segment *AD*. What is the area of parallelogram *ABCD*?



Note: Figure not drawn to scale

Answer: 24

The area of a parallelogram is A = base × height. If BC has length 8, then AD also has length 8. If AD is the base, then BE can be the height, because base and height must be perpendicular. We are told BE bisects AD, so AE must have length 4. We see then that triangle ABE is a 3–4–5 triangle in disguise. BE has length 3, and the area is $(8)(3) = 24$.

Quantity A	Quantity B
$(3)(5)$	$3 + 5$

Answer: (A) Quantity A is greater.

Quantity A: $(3)(5) = 15$

Quantity B: $3 + 5 = 8$

Quantity A	Quantity B
$3(5 + 1)$	$(3)(5) + (3)(1)$

Answer: (C) The two quantities are equal.

Quantity A: $3(5 + 1) = 3(6) = 18$

Quantity B: $(3)(5) + (3)(1) = 15 + 3 = 18$

Alternatively, note that Quantity B is the distributed form of Quantity A, or Quantity A is the factored form of Quantity B.

Quantity A	Quantity B
$3(5 + 1)$	$(3)(5)(1)$

Answer: (A) Quantity A is greater.

Quantity A: $3(5 + 1) = 3(6) = 18$

Quantity B: $(3)(5)(1) = 15$

Or, note that both quantities are 3 times something, and that $5 + 1 > 5$.

Quantity A	Quantity B
$3(5 + 2)$	$3 \div \left(\frac{1}{5} + \frac{1}{2}\right)$

Answer: (A) Quantity A is greater.

Quantity A: 3(5 + 2) = 3(7) = 21

Quantity B:

$$3\div\left(\frac{1}{5}+\frac{1}{2}\right)=3\div\left(\frac{2}{10}+\frac{5}{10}\right)=3\div\left(\frac{7}{10}\right)$$
$$=3\times\left(\frac{10}{7}\right)=\frac{30}{7}=4\frac{2}{7}$$

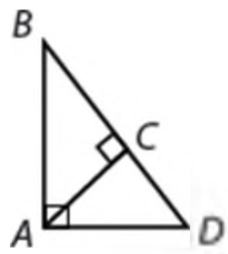
Or, note that both quantities are 3 times something. We can compare 7 to the reciprocal of $\left(\frac{1}{5}+\frac{1}{2}\right)$, which will be less than than 7, since the fraction sum is greater than than 1/7.

Quantity A	Quantity B
47% of 1,200	153% of 400

Answer: (B) Quantity B is greater.

Quantity A: 47% of 1,200 is a bit less than half of 1,200. Less than 600.

Quantity B: 153% of 400 is a bit more than 1.5 times 400. More than 600.



$AB = 2AD$

Note: Figure not drawn to scale

Quantity A	Quantity B
BC	CD

Answer: (A) Quantity A is greater.

Drawing the figure to scale helps quite a bit:

BC is longer than *CD* by a significant margin; you can trust the to-scale picture.

AC splits right triangle *ABD* into two more similar right triangles: *ABC* and *ACD*. In similar triangles, the ratio of short leg to the long leg is constant (here, a ratio of 1:2). In *ABC*, *BC* is the long leg and *AC* is the short leg. In *ACD*, *AC* is the long leg and *CD* is the short leg. So, $BC > AC > CD$.

$y = 3x^2 - 14x + 5$

<u>**Quantity A**</u> | <u>**Quantity B**</u>

x | y

Answer: (D) The relationship cannot be determined from the information given

Try:

If $x = 0$, $y = 0 - 0 + 5 = 5$. Since $5 > 0$, $y > x$.

If $x = 1$, $y = 3 - 14 + 5 = -6$. Since $1 > -6$, $x > y$.

$a > 3$

Quantity A	Quantity B
$\dfrac{5+2a}{3}$	a

Answer: (D) The relationship cannot be determined from the information given

Algebra solution:
$$\begin{aligned} \frac{5+2a}{3} &\ ?\ a \\ 5+2a &\ ?\ 3a \\ 5 &\ ?\ a \end{aligned}$$

If $a > 3$, a could be less than, equal to, or greater than 5.

Number testing solution:

When $a = 4$, $\frac{5+2a}{3} = \frac{5+8}{3} = \frac{13}{3} = 4.\overline{3}$. Quantity A is greater.

When $a = 6$, $\frac{5+2a}{3} = \frac{5+12}{3} = \frac{17}{3} = 5.\overline{6}$. Quantity B is greater.

Quantity A	Quantity B
$\dfrac{7!}{4!}$	$\dfrac{6!}{3!}$

Answer: (A) Quantity A is greater.

Quantity A:

$$\frac{7!}{4!}=\frac{(7)(6)(5)(4)(3)(2)(1)}{(4)(3)(2)(1)}=(7)(6)(5)=210$$

Quantity B: $\frac{6!}{3!}=\frac{(6)(5)(4)(3)(2)(1)}{(3)(2)(1)}=(6)(5)(4)=120$

Or, notice that both simplified products include (6)(5). 7 times that product is greater than 4 times that product.

Quantity A	Quantity B
$\dfrac{6!}{4!2!}$	$\dfrac{5!}{3!}$

Answer: (B) Quantity B is greater.

Quantity A: $\frac{6!}{2!4!} = \frac{(6)(5)(4)(3)(2)(1)}{(2)(1)(4)(3)(2)(1)} = \frac{(6)(5)}{(2)(1)} = 15$

Quantity B: $\frac{5!}{3!} = \frac{(5)(4)(3)(2)(1)}{(3)(2)(1)} = (5)(4) = 20$

Quantity A	Quantity B
$x^2 + 4$	$4x$

Answer: (D) The relationship cannot be determined from the information given.

$$
\begin{array}{lcc}
x^2 + 4 & ? & 4x \\
x^2 - 4x + 4 & ? & 0 \\
(x-2)(x-2) & ? & 0 \\
(x-2)^2 & ? & 0
\end{array}
$$

As a squared term, $(x - 2)^2$ can never be less than 0. But $(x - 2)^2$ can equal 0 when $x = 2$, or $(x - 2)^2$ can be greater than 0 when $x \neq 2$.

You could test numbers, but if you did not test $x = 2$, you might incorrectly conclude that Quantity A is always greater.

$m < 1$

Quantity A | **Quantity B**

$m + 1$ | $3m - 2$

Answer: (A) Quantity A is greater.

$$
\begin{array}{ccc}
m+1 & ? & 3m-2 \\
m+3 & ? & 3m \\
3 & ? & 2m \\
3/2 & ? & m \\
\hline
\end{array}
$$

$$3/2 > 1 > m$$

m must be less than 1, which is less than 3/2. Quantity A must be greater.

What is x?

$$7x + 10 = 4x - 5$$

Answer: $x = -5$

$7x + 10 = 4x - 5$

$3x + 10 = -5$

$3x = -15$

$x = -5$

If $x \neq 0$, Which of the following is closest to $-x$ on the number line?

(A) 0

(B) x

(C) $-\frac{3}{2}x$

(D) $\frac{1}{3}x$

(E) $-\frac{2}{3}x$

Answer: (E) $-\frac{2}{3}x$

Pick a smart number for x. If $x = -6$, we are looking for the answer that is closest to $+6$ on the number line.

(A) 0. Distance = 6

(B) $x = -6$. Distance = 12

(C) $-\frac{3}{2}x = 9$. Distance = 3

(D) $\frac{1}{3}x = -2$. Distance = 8

(E) $-\frac{2}{3}x = 4$. Distance = 2

Any other smart number would give us the same result—(E) is smallest.

$$x = -1$$

Quantity A	Quantity B
x^2	$x - 1$

Answer: (A) Quantity A is greater.

Quantity A: When $x = -1$, x^2 is positive (specifically, +1).

Quantity B: When $x = -1$, $x - 1$ is negative (specifically, −2).

$x = -1$

Quantity A	Quantity B
$2x + 1$	$-2x + 1$

Answer: (B) Quantity B is greater.

Calculate, avoiding silly errors:

Quantity A: $2x + 1 = -2 + 1 = -1$

Quantity B: $-2x + 1 = 2 + 1 = 3$

$$x = -1$$

Quantity A	Quantity B
$3x - 1$	$2x + 1$

Answer: (B) Quantity B is greater.

Calculate, avoiding silly errors:

Quantity A: $3x - 1 = -3 - 1 = -4$

Quantity B: $2x + 1 = -2 + 1 = -1$

$x = -1$

Quantity A	Quantity B
$2x - 1$	$3x + 1$

Answer: (B) Quantity B is greater.

Calculate, avoiding silly errors:

Quantity A: $2x - 1 = -2 - 1 = -3$

Quantity B: $3x + 1 = -3 + 1 = -2$ (greater than -3)

Which two of the following numbers have a product that is greater than 10?

(A) −6

(B) −3

(C) 2

(D) 4

Answer: (A) and (B)

$(A)(B) = (-6)(-3) = 18$

For a positive product of two terms, we need to pair either the two positive choices, $(2)(4)$, or the two negative choices, $(-6)(-3)$. The product of the two negative choices is greater than 10.

Which two of the following numbers have a product that is less than -15?

(A) -6

(B) -3

(C) 2

(D) 4

Answer: (A) and (D)

$(A)(D) = (-6)(4) = -24$

For a negative product of two terms, we need to pair a positive choice with a negative choice. There ares four possibilities to check, but since the answer must be *less than* a certain value, it makes sense that it results from the smallest negative choice (-6) times the greatest positive choice (4).

Which <u>three</u> of the following numbers have a product that is greater than 40?

[A] −6

[B] −3

[C] 2

[D] 4

Answer: [A], [B], and [D]

$[A][B][D] = (-6)(-3)(4) = 72$

For a positive product of three terms, we need either (pos)(pos)(pos), which is not possible with the choices offered, or (neg)(neg)(pos). Thus, we must include [A] and [B], and we maximize the product by selecting the greatest positive remaining choice, [D].

Which <u>three</u> of the following numbers have a product that is less than -30?

[A] -6

[B] -3

[C] 2

[D] 4

Answer: [A], [C], and [D]

$[A][C][D] = (-6)(2)(4) = -48$

For a negative product of three terms, we need either (neg)(neg)(neg), which is not possible with the choices offered, or (neg)(pos)(pos). Thus, we must include [C] and [D], and we minimize the product by selecting the most negative remaining choice, [A].

What is the units digit of 72^{48}?

Answer: 6

In a product (which is fundamentally what an exponent represents), to find the units digit, only pay attention to the units digits of the numbers you are working with. Drop any other digits. Thus, only consider the 2 in the units digit of 72. The powers of 2 are: **2**, **4**, **8**, 1**6**, 3**2**, 6**4**, 12**8**, 25**6**, 51**2**, etc. Notice that the units digits repeat in a 4-term cycle: [2, 4, 8, 6]. The 48^{th} power is at the end of the twelfth 4-term repeat cycle, so 72^{48} has a units digit of 6.

The cost of item A is \$0.17, and the cost of item B is \$0.25. What is the total cost of 50 item A's and 21 item B's, in dollars?

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Answer: \$13.75

$50(\$0.17) + 21(\$0.25) = \$8.50 + \$5.25 = \$13.75$

Alternatively, some "mental math":

Half of $100(\$0.17)$ + 21 quarters = Half of \$17 + \$5.25 = \$8.50 + \$5.25

If a group of 10 students includes exactly six seniors and exactly five women, what are the minimum and maximum possible numbers of women seniors?

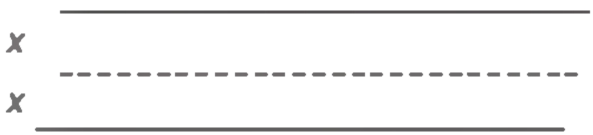
Answer: Maximum = 5, Minimum = 1

Maximum: Occurs if all of the women are seniors. Note that not all of the seniors be women, as there are fewer women than seniors in the group. Group = 5 senior women, 1 senior man, and 4 non-senior men.

Minimum: Occurs when the number of women non-seniors is maximized. There are 4 non-seniors in the group, all of which could be women, leaving 1 woman who must be a senior. Group = 5 senior men, 1 senior woman, and 4 non-senior women.

How many lines in the plane are equidistant from the parallel lines above?

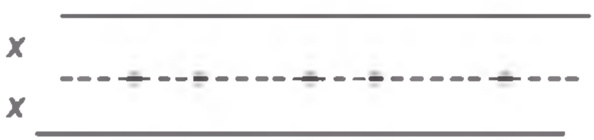
Answer: 1 line, halfway between the two parallel lines

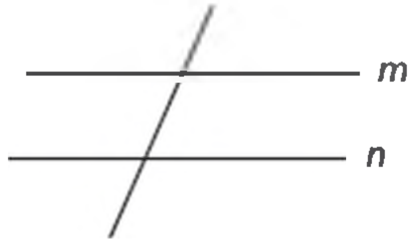


How many points in the plane are equidistant from the parallel lines above?

Answer: An infinite number of points

Any point on the dotted line below is equidistant from the parallel lines. Some example points are shown below.



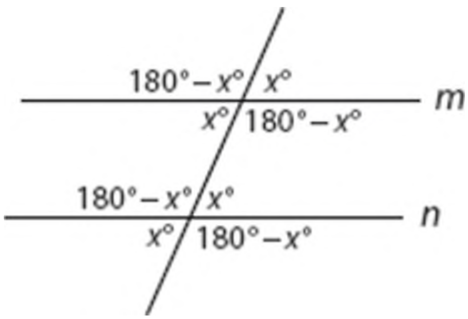


In the figure above, lines m and n are parallel.

In order to determine all of the angles above, what information would suffice?

Answer: Any one of the angles

Knowing any one of the angles would allow us to determine all the angles. In such a figure (parallel lines cut by a transversal), the relationship among all eight angles is known:



For example, if $x = 55$, you could label all of the angles as either 55° or 125°, according to the figure above.

Simplify:

$$a^{m} \times a^{n-m}$$

Answer: a^n

When multiplying terms that have a common base, add the exponents:

$$a^m \times a^{n-m} = a^{m+n-m} = a^n$$

Simplify:

$$3^3 + 3^3 + 3^3 + 3^3 + 3^3 + 3^3 + 3^3 + 3^3 + 3^3$$

Answer: 3^5

$$\begin{aligned} 3^3 + 3^3 + 3^3 + 3^3 + 3^3 + 3^3 + 3^3 + 3^3 + 3^3 &= 9(3^3) \\ &= 3^2(3^3) \\ &= 3^{2+3} \\ &= 3^5 \end{aligned}$$

x and y are positive integers such that $xy = 16$

Quantity A	Quantity B
The absolute difference between x and y	0

Answer: (D) The relationship cannot be determined from the information given.

If x and y are both positive such that $xy = 16$, there are the following possibilities:

x	y	Absolute difference = $\|x - y\|$
1	16	15
2	8	6
4	4	0
8	2	6
16	1	15

The absolute difference between x and y can be 15, 6, or 0. Do not assume, just because the variables are different letters, that x cannot equal y! x might equal y in this case, or it might not.

Quantity A	Quantity B
Two times the average of x, y, and z	The average of $2x$, $2y$, and $2z$

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Answer: (C) The two quantities are equal.

Quantity A: $2\times\left(\frac{x+y+z}{3}\right)=\frac{2}{3}(x+y+z)$

Quantity B: $\frac{2x+2y+2z}{3}=\frac{2(x+y+z)}{3}=\frac{2}{3}(x+y+z)$

If the positive integer x is divisible by 6, is x divisible by 3?

Answer: Yes

A number is always divisible by any of its factors. Since x is divisible by 6, x has the factors of 6, including 2 and 3. Thus, x is divisible by both 2 and 3.

For example: x is divisible by 6, so x could be 6, 12, 18, 24, 30, 36, etc.

All of the possible values of x are divisible by 3.

If the positive integer x is divisible by 6, is x divisible by 12?

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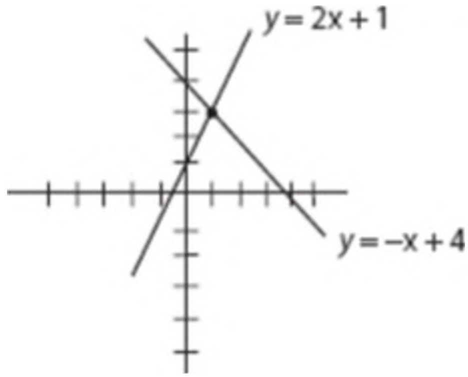
Answer: Maybe

In order for x to be divisible by 12, it would need to have the factors of 12: (2)(2)(3). Since x is divisible by 6, it has the necessary factor of 3 and one of the necessary factors of 2. However, it is uncertain whether x has the second necessary factor of 2.

For example: x is divisible by 6, so x could be 6 or 24.

24 is divisible by 12, but 6 is not.

If $y = 2x + 1$ and $y = -x + 4$, solve for x and y.



Answer: $x = 1$ and $y = 3$

A point being on a line is the same as the equation "working" when you plug in the (x, y) coordinates for that point. For example, the points (1, 3) and (3, 7) are both solutions for $y = 2x + 1$, as both points are on that line. So for a solution to "work" for two equations, it would have to be a point lying on both lines. Here, the only such point is the intersection of the two lines, at (1, 3).

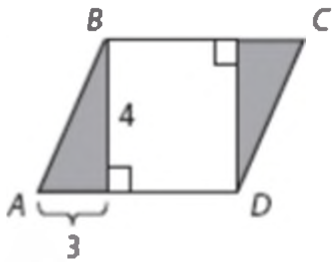
Algebra: Since both $2x + 1$ and $-x + 4$ equal y, they equal each other.

$2x + 1 = -x + 4$

$3x = 3$

$x = 1$

Plug $x = 1$ into either equation to get $y = 3$.



Note: Figure not drawn to scale

Quantity A	Quantity B
The shaded area of rhombus *ABCD*.	The non-shaded area of rhombus *ABCD*.

Answer: (A) Quantity A is greater.

Notice that the two shaded triangles are both 3–4–5 right triangles.

$$\text{Shaded area} = 2\times\left(\frac{1}{2}bh\right) = 2\times\left(\frac{1}{2}\times 3\times 4\right) = 12$$

If sides *AB* and *CD* have length *S*, then so must *BC* and *AD* (because *ABCD* is a rhombus). Therefore the non-shaded portions of *BC* and *AD* have length 2.

The non shaded area is a rectangle: $lw = 4 \times (5 - 3) = 8$

Which of the following is greatest?

(A) $1 + 2 - 3 + 4 - 5 + 6 - 7 + 8$

(B) $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8$

(C) $1 - 2 - 3 - 4 - 5 + 6 + 7 + 8$

(D) $1 + 2 + 3 + 4 - 5 - 6 - 7 - 8$

Answer: (C)

Of course, you could just compute.

(A) $1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 = 6$

(B) $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 = -4$

(C) $1 - 2 - 3 - 4 - 5 + 6 + 7 + 8 = 8$

(D) $1 + 2 + 3 + 4 - 5 - 6 - 7 - 8 = -16$

Tedious calculations can lead to silly errors, so strategic elimination is a good double-check.

(B) Pair terms: $(1 - 2) + (3 - 4) + (5 - 6) + (7 - 8)$ = sum of four negatives < 0

(D) Pair terms: $(1 - 5) + (2 - 6) + (3 - 7) + (4 - 8)$ = sum of four negatives < 0

In (A) and (C), the negative terms are generally smaller in absolute value than the positive terms, so expect positive results and be most careful in calculating (A) and (C).

If 35 percent of the rooms at a hotel have an ocean view, what is the ratio of the number of rooms with an ocean view to the number of rooms without?

Answer: $\frac{7}{13}$

$$\frac{\text{With}}{\text{Without}} = \frac{35\%}{100\% - 35\%} = \frac{35\%}{65\%} = \frac{35}{65} = \frac{7}{13}$$

What is 135% of 140?

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Answer: 189

100% of $140 = 140$

10% of $140 = 14$

30% of $140 = 3(14) = 42$

5% of $140 = 14/2 = 7$

So, 135% of 140 is $140 + 42 + 7 = 189$.

Alternatively, calculate $1.35 \times 140 = 189$.

What is the sum of all the integers from 11 to 30, inclusive?

Answer: 410

Sum of consecutive integers = $\frac{\text{First} + \text{Last}}{2} \times \text{\# of Integers}$

The number of integers is First – Last + 1 = 30 – 11 + 1 = 20.

$$\text{Sum} = \frac{11+30}{2} \times 20 = 41 \times 10 = 410.$$

Alternatively, with a calculator: 11 + 12 + 13 + 14 +...+ 27 + 28 + 29 + 30, but even with a calculator, this method is more time consuming and error-prone.

Quantity A	Quantity B
$0.5+\frac{1}{2}+\frac{10}{5}$	$0.7+\frac{1}{3}+2$

Answer: (B) Quantity B is greater.

Note that the last term in each sum is +2. Ignore 10/5 in Quantity A and ignore 2 in Quantity B.

Quantity A: $0.5+\frac{1}{2}=\frac{1}{2}+\frac{1}{2}=1$

Quantity B: $0.7+\frac{1}{3}=\frac{7}{10}+\frac{1}{3}=\frac{21}{30}+\frac{10}{30}=\frac{31}{30}>1$

Alternatively, use decimals for Quantity B: $0.7+\frac{1}{3}=0.7+0.333=1.033>1$

3 rabbits each weigh an average of 8 pounds. What would a 4^{th} rabbit need to weigh in order to reduce the average weight of the 4 rabbits to 7.5 pounds?

185

Answer: 6 pounds

Sum = *Average* × *# of terms*, so the total weight of the first 3 rabbits is $8 \times 3 = 24$ pounds.

If x is the weight of the 4^{th} rabbit, and the average weight of all 4 is to be 7.5 pounds,

$$7.5 = \frac{24+x}{4} \rightarrow 30 = 24 + x \rightarrow 6 = x$$

Alternatively, consider that the 4^{th} rabbit only has 1/4 "influence" on the average weight of all 4 rabbits, yet is reducing the average weight by 0.5 pounds. To do so, it would have to be 4(0.5 pounds) = 2 pounds lighter than the average weight of the other rabbits.

Quantity A	Quantity B
$(-15)^2(12)^3$	$(15)^3(-12)^3$

Answer: (A) Quantity A is greater.

Quantity A: $(-15)^2(12)^3$ = positive × positive = positive

Quantity B: $(15)^3(-12)^3$ = positive × negative = negative

Simplify: $\dfrac{0.37+\dfrac{1}{4}}{\dfrac{31}{25}}$

Answer: $\frac{1}{2}$

$$\frac{0.37+\frac{1}{4}}{\frac{31}{25}}=\frac{0.37+0.25}{\frac{31}{25}}=\frac{0.62}{\frac{31}{25}}=\frac{\frac{62}{100}}{\frac{31}{25}}=\frac{62}{100}\times\frac{25}{31}=\frac{2}{4}=\frac{1}{2}$$

Quantity A	Quantity B
$3(7^2 + 11^2) + 2^5$	$147 + (121)(3) + 4^2$

Answer: (A) Quantity A is greater.

Only do enough math to recognize similarities between the two quantities.

Quantity A: $3(7^2 + 11^2) + 2^5 = 3(49 + 121) + 2^5$

Quantity B: $147 + (121)(3) + 4^2 = 3(49 + 121) + (2^2)^2$

Only the last additive term is different, and $2^5 > 2^4$.

What is 160% of 45?

189

Answer: 72

100% of 45 = 45

10% of 45 = 4.5

60% of 45 = 6(4.5) = 27

So, 160% of 45 is 45 + 27 = 72.

Alternatively, calculate $1.6 \times 45 = 72$.

Which of the following is the graph on the number line for all values of x such that $x^3 < x^2$?

(A) 

(B) 

(C) 

(D) 

(E) 

Answer: (B)

You could try values in the different ranges shown:

$x = -3$ works: $-27 < 9$

$x = -1$ works: $-1 < 1$

$x = -1/2$ works: $-1/8 < 1/4$

$x = 0$ does NOT work: $0 = 0$

$x = 1/2$ works: $1/8 < 1/4$

$x = 1$ does NOT work: $1 = 1$

$x = 3$ does NOT work: $27 > 9$

Algebra solution: $x^3 < x^2$ is true when $x^3 - x^2 < 0$, or $x^2(x - 1)$ is negative.

x^2 is positive for non-zero x values. $x - 1$ will be negative when $x < 1$. So generally, $x < 1$ is the range where $x^3 < x^2$. However, $x^2(x - 1)$ equals 0 when $x = 0$ or 1, so these values are not allowed.

Simplify: $\sqrt{5^6}$

191

Answer: 5^3 or 125

$$\sqrt{5^6} = \left(5^6\right)^{1/2} = 5^{6 \times \frac{1}{2}} = 5^3$$

Alternatively, remember why this rule works:

$$\sqrt{5^6} = \sqrt{(5)(5)(5)(5)(5)(5)} = \left(\sqrt{5 \times 5}\right)\left(\sqrt{5 \times 5}\right)\left(\sqrt{5 \times 5}\right) = 5 \times 5 \times 5 = 5^3$$

This process works perfectly with numbers. With variables, you have to put in absolute value signs to force the outcome to be positive: $\sqrt{x^6} = |x^3|$.

Simplify: $\frac{368^0}{\sqrt{16}}$

192

Answer: $\frac{1}{4}$

Any non-zero number to the 0^{th} power equals 1.

$$\frac{368^0}{\sqrt{16}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$$

Simplify: $\frac{2}{7}+\frac{5}{3}$

Answer: $\frac{41}{21}$

$$\frac{2}{7}+\frac{5}{3}=\frac{3}{3}\left(\frac{2}{7}\right)+\frac{7}{7}\left(\frac{5}{3}\right)=\frac{6}{21}+\frac{35}{21}=\frac{41}{21}$$

Simplify: $(3x^3)^2$

194

Answer: 3^2x^6 or $9x^6$

$(3x^3)^2 = (3^1x^3)^2 = (3^{1 \times 2})(x^{3 \times 2}) = 3^2x^6$ or $9x^6$

Simplify: $\frac{1}{4} \div \frac{1}{3}$

Answer: $\frac{3}{4}$

$$\frac{1}{4} \div \frac{1}{3} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$$

Factor: $x^2 + 5x - 6$

Answer: $(x + 6)(x - 1)$

6 can be factored as either $(3)(2)$ or $(6)(1)$. To achieve a middle term of $5x$, note that 3 and 2 *sum* to 5, whereas 6 and 1 *differ* by 5. Since the last term is -6, the factor numbers must have different signs, so pick the factor pair that *differs* by 5: 6 and -1 are the factor numbers.

$x^2 + 5x - 6 = (x + 6)(x - 1)$

Given $x \neq -4$, simplify:

Answer: $x - 4$

Recognize the "difference of two squares" common quadratic:

$$\frac{x^2-16}{x+4}=\frac{(x+4)(x-4)}{x+4}=x-4$$

Alternatively, express 2.25 as a fraction:

$$2.25=2\frac{1}{4}=2+\frac{1}{4}=\frac{8}{4}+\frac{1}{4}=\frac{9}{4}$$

Then flip $\frac{9}{4}$ to get $\frac{4}{9}$.

What is $\frac{1}{3}+0.87+\frac{11}{50}$ in decimal form, rounded to the nearest 0.01?

Answer: 1.42

$$\begin{aligned}\frac{1}{3}+0.87+\frac{11}{50}&=0.33\overline{3}+0.87+\frac{22}{100}\\&=0.33\overline{3}+0.87+0.22\\&=1.42\overline{3}\end{aligned}$$

Because the thousandths digit is 4 or less, round down to the nearest 0.01.

What is the reciprocal of 2.25?

Answer: $\frac{4}{9}$

The reciprocal of 2.25 is $\frac{1}{2.25} = \frac{1}{225/100} = \frac{100}{225} = \frac{4}{9}$

What is the reciprocal of 3.5?

Answer: $\frac{2}{7}$

The reciprocal of 3.5 is $\frac{1}{3.5}=\frac{1}{7/2}=\frac{2}{7}$.

Alternatively, express 3.5 as a factor:

$$3.5=3\frac{1}{2}=3+\frac{1}{2}=\frac{6}{2}+\frac{1}{2}=\frac{7}{2}$$

Then flip $\frac{7}{2}$ to get $\frac{2}{7}$.

What is the reciprocal of 0.4?

201

Answer: $\frac{5}{2}$ or 2.5

The reciprocal of 0.4 is $\frac{1}{0.4}=\frac{1}{4/10}=\frac{10}{4}=\frac{5}{2}=2.5$.

Alternatively, express 0.4 as a fraction:

$$0.4=\frac{4}{10}=\frac{2}{5}$$

Then flip $\frac{2}{5}$ to get $\frac{5}{2}$.

What is the remainder when 1,827 is divided by 4?

Answer: The remainder is 3.

A number is divisible by 4 if the two-digit number at the end is divisible by 4. For example, 367 is not divisible by 4 because 67 is not divisible by 4. In contrast, 360 is divisible by 4 because 60 is divisible by 4.

So 1,827 is not divisible by 4, but 1,824 and 1,828 would be. Thus, when 1,827 is divided by 4, the remainder is $1{,}827 - 1{,}824 = 3$.

In other words, we can drop everything but the last two digits:

$1{,}827 \longrightarrow$ 27 gives a remainder of 3 when divided by 4.

What is the remainder when 18,271 is divided by 3?

Answer: The remainder is 1.

A number is divisible by 3 if the sum of its digits is divisible by 3. For 18,271, the sum of the digits is $1 + 8 + 2 + 7 + 1 = 19$. This is 1 more than a multiple of 3. If 18,271 were 1 less (i.e. 18,270), it would be a multiple of three. Thus, when 18,271 is divided by 3, the remainder is 1.

In addition to checking for whether a number is divisible by 3, you can also use the sum of the digits to determine the remainder when dividing by 3. Since the digit sum 19 divided by 3 gives a remainder of 1, so does the original number 18,271.

Given $xy \neq 0$, simplify: $\frac{81x^3y^4}{27(xy)^2}$

Answer: $3xy^2$

$$\frac{81x^3y^4}{27(xy)^2}=\frac{81}{27}\times\frac{x^3y^4}{x^2y^2}=3x^{3-2}y^{4-2}=3xy^2$$

Factor: $x^4 + 2x^3 + x^2$

Answer: $x^2(x+1)^2$

$$\begin{aligned}
x^4+2x^3+x^2 &= x^2\left(x^2+2x^1+x^0\right)\\
&= x^2\left(x^2+2x+1\right)\\
&= x^2(x+1)(x+1)\\
&= x^2(x+1)^2
\end{aligned}$$

The price of a jacket was first reduced 20%, then was increased back to 90% of the original price. If the original price of the jacket was x, which of the following represents the amount of the most recent price increase?

(A) $(0.1)(0.2)x$

(B) $(0.8)(0.9)x$

(C) $(0.8)(1.1)x$

(D) $(0.9 - 0.8)x$

(E) $(1\text{-}(0.8)(1.1))x$

Answer: (D)

A reduction of 20% can be expressed as times $(1 - 0.2)$, or simply times 0.8. So, after "the price of a jacket was first reduced 20%," it cost $0.8x$.

"90% of the original price" is $0.9x$.

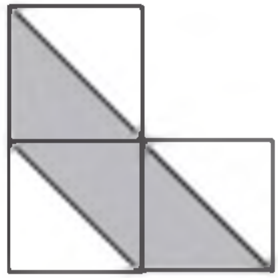
So the most recent price increase was from $0.8x$ to $0.9x$, or an increase of $0.9x - 0.8x = (0.9 - 0.8)x$, or $0.1x$.

Simplify: $\frac{3^x}{3^{2x-1}}$

207

Answer: 3^{-k+1} or 3^{1-k}

$$\frac{3^k}{3^{2k-1}} = 3^{k-(2k-1)} = 3^{k-2k-(-1)} = 3^{-k+1} = 3^{1-k}$$



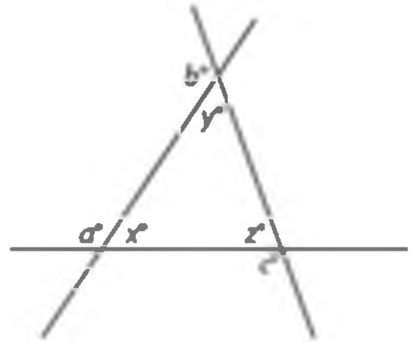
Each of the three squares above has side length of 8.

Quantity A	Quantity B
The area of the shaded region.	96

Answer: (C) The two quantities are equal.

The diagonal lines in the figure split each square exactly in half, creating 6 identical right triangles with legs of length 8. The shaded area is 3 triangles, and the unshaded area is 3 triangles. Thus, the shaded area is half the area of the 3 squares.

Quantity A: $3\times\frac{(8)(8)}{2}=3\times\frac{64}{2}=3\times 32=96$



<u>Quantity A</u>	**<u>Quantity B</u>**
The average of x, y, and z.	(The average of a, b, and c) ÷ 2

Answer: (C) The two quantities are equal.

Quantity A: $\frac{x+y+z}{3}=\frac{180}{3}=60$

An exterior angle of a triangle is equal in measure to the sum of the two opposite interior angles of the triangle, so the average of *a*, *b*, and *c* is

$$\frac{a+b+c}{3}=\frac{(y+z)+(x+z)+(x+y)}{3}=\frac{2(x+y+z)}{3}=\frac{2(180)}{3}=120$$

Quantity B: $120 \div 2 = 60$

Jeremy cut a 64 foot rope into 5 pieces. If each piece is at least 10 feet long, what is the longest any of the 5 pieces could be?

Answer: 24 feet

To maximize the length of one piece, minimize the length of the other four. In other words, if four of the pieces were minimized to 10 feet long, all the "extra" length would be assigned to the one remaining piece.

$64 - 4(10) = 64 - 40 = 24$

If it takes 5 machines 1 hour to complete 3 job lots, how long will it take 2 machines to complete 2 job lots?

Answer: $\frac{5}{3}$ hours, or 100 minutes.

Rather than setting up equations with variables, it may be easier to reason through the given information step-by-step.

5 machines complete 3 lots in 1 hour.

1 machine completes 3/5 lot in 1 hour.

1 machine completes 1 lot in 5/3 hour.

2 machines complete 2 lots in 5/3 hour.

A certain substance reduces in mass by half every 20 years. If the mass 100 years ago was 3,200 milligrams, how many years from now will the mass be 25 milligrams?

Answer: 40 years from now

Set up a Population Chart:

Year	milligrams
−100	3,200
−80	1,600
−60	800
−40	400
−20	200
Now	100
+20	50
+40	25

At a certain school, there are 24 students per teacher and 2 administrative staff members for every 3 teachers. If there are 6 administrative staff members, how many students attend the school?

Answer: 216

Admin : Teachers : Students = $2 : 3 : (3)(24) = 2 : 3 : 72$

If there are 6 administrative staff members, the ratio above must be multiplied by 3. Remember to multiply each term in the ratio, so their relationships to one another don't change.

$2(3) : 3(3) : 72(3) = 6 : 9 : 216$

Simplify: $4^4 + 4^4 + 4^4 + 4^4$

214

Answer: 4^5 or 2^{10} or 1,024

$$4^4 + 4^4 + 4^4 + 4^4 = 4(4^4) = 4^1(4^4) = 4^{1+4} = 4^5$$

Since 4 is not a prime number, an equivalent alternative is:

$$4^5 = (2^2)^5 = 2^{2 \times 5} = 2^{10}$$

If you know your powers of 2 (or use the calculator), you know that 2^{10} works out to be 1,024, or approximately 1,000. A "kilobyte" is actually 1,024 bytes of data.

Simplify: $4^4 + 4^4$

215

Answer: 2^9 or 512

$$4^4+4^4=2\left(4^4\right)=2^1\left(\left(2^2\right)^4\right)=2^1\left(2^{2\times4}\right)=2^1\left(2^8\right)=2^{1+8}=2^9$$

If you know your powers of 2 (or use the calculator), you know that 2^9 works out to be 512.

If $z \neq 0$, simplify $\frac{z^{-2q}z^{2q-1}}{z}$.

216

Answer: 1

$$\frac{z^{-2q}z^{2q+1}}{z}=\frac{z^{(-2q)+(2q+1)}}{z}=\frac{z^{1}}{z}=1$$

What is the median of 25, 26, 29, 99, and 501?

What is the average (arithmetic mean) of 25, 26, 29, 99, and 501?

Answer: Median = 29, Mean = 136

The median is the middle number in the set when arranged from least to greatest. In this set of 5 terms, the median is the 3^{rd} largest (or, equivalently, the 3^{rd} smallest).

$$\text{Average} = \frac{25+26+29+99+501}{5} = \frac{680}{5} = 136$$

$$ab < 0 \text{ and } \frac{4}{a} > \frac{-2}{b}$$

<u>Quantity A</u>	<u>Quantity B</u>
$4b$	$-2a$

Answer: (B) Quantity B is greater.

Multiply both sides by *ab* to eliminate the denominators. Since *ab* is negative, this requires the inequality direction to be flipped.

$$ab\left(\frac{4}{a}\right) < \left(\frac{-2}{b}\right)ab$$
$$4b < -2a$$

$$y \neq 0 \text{ and } x > 0 \text{ and } \frac{2}{y^3} > \frac{7}{x}$$

Quantity A	Quantity B
$2x$	$7y^2$

Answer: (A) Quantity A is greater.

Since $y \neq 0$, y^2 is positive. The problem also states that x is also positive. So multiply both sides by y^2x to eliminate the denominators, and there is no need to flip the sign of the inequality.

$$y^2x\left(\frac{2}{y^2}\right) > \left(\frac{7}{x}\right)y^2x$$
$$2x > 7y^2$$

Even or Odd?

Even × Odd + Odd × Odd = ____

Answer: Odd

Order of operations is PEMDAS: **P**arentheses, **E**xponents, **M**ultiplications/**D**ivision, and **A**ddition/**S**ubtraction. So do the multiplication in this problem before the addition. Parentheses can serve as a visual reminder of what to do first:

(Even × Odd) + (Odd × Odd) = Even + Odd = Odd

Simplify:

221

Answer: 3

$$81^{1/4} = \left(81^{1/2}\right)^{1/2} = \sqrt{\sqrt{81}} = \sqrt{9} = 3$$

Simplify: $\sqrt[5]{1{,}024}$

Answer: 4

1,024 is a number you should recognize, as it is a power of 2.

$1{,}024 = 2^{10}$

$$\sqrt[5]{1{,}024} = 1{,}024^{1/5} = \left(2^{10}\right)^{1/5} = 2^{10\times 1/5} = 2^2 = 4$$

12 is what percent of 1,000?

223

Answer: 1.2%

Translate and solve:

$$12 = \frac{x}{100} \times 1{,}000$$

$$12 = 10x$$

$$x = 1.2$$

Alternatively, 1% of 1,000 is 10. So, because 12 is 1.2 times 10, the answer is 1.2 times 1%, or 1.2%.

$$\begin{array}{r} 89{,}264 \\ +19{,}735 \\ \hline \end{array}$$

Answer: 108,999

Use the calculator or compute on paper. For the latter, add vertically in each digit place, starting at the right side (the units digit). Don't forget to "carry the 1" when a digit sum is greater than 9.

Also, you should "sanity check" answers you get even when using the calculator, as numbers can easily be mis-entered. Just over 89,000 plus almost 20,000 should be about 109,000.

Quantity A	Quantity B
$2^6 3^5$	$2^7 5^3$

Answer: (B) Quantity B is greater.

2^6 is a factor of both quantities, so cancel/ignore it when comparing.

Quantity A: $3^5 = 243$

Quantity B: $2^1\ 5^3 = 2 \times 125 = 250$

Quantity A	Quantity B
$2^4 4^2$	$2^5 5^2$

226

Answer: (B) Quantity B is greater.

The first term in each product has a base of 2. The exponent in Quantity B is greater, so $2^5 > 2^4$.

The second term in each product has an exponent of 2. The base in Quantity B is greater, so $5^2 > 4^2$.

Quantity B is the product of two larger terms than in Quantity A.

Alternatively, ignore the common factor of 2^4:

Quantity A: $4^2 = 16$
Quantity B: $2 \times 5^2 = 50$

Quantity A	Quantity B
3^8	9^4

Answer: (C) The two quantities are equal.

Put Quantity B in terms of a prime base.

Quantity B: $9^4 = (3^2)^4 = 3^{2\times4} = 3^8$

General rule: Squared base and halved exponent means equal. ($9 = 3^2$ and $4 = \frac{1}{2} \times 8$)

What percent of 4 is 6?

228

Answer: 150%

Translate and solve:

$$\frac{x}{100}\times 4=6$$

$$x=6\times\frac{100}{4}$$

$$x=150$$

Put the number answer back into the question to check: 150% of 4 is 6.

Alternatively, think: 6 is 4 plus half of 4, so 6 is 100% + 50% of 4, or 150% of 4.

Quantity A

$$\frac{17}{4} \times \frac{3}{5} \times \frac{2}{3}$$

Quantity B

$$\frac{17}{4} + \frac{3}{5} + \frac{2}{3}$$

Answer: (B) Quantity B is greater.

Both quantities start with $\frac{17}{4}$. In Quantity A, this is multiplied by a positive fraction less than 1, reducing the value, and then again. In Quantity B, this is increased by adding a positive fraction, and then again. The exact values don't matter: Quantity B > $\frac{17}{4}$ > Quantity A.

Quantity A	Quantity B
11% of 37	37% of 11

Answer: (C) The two quantities are equal.

Quantity A: $\frac{11}{100} \times 37 = \frac{11 \times 37}{100}$

Quantity B: $\frac{37}{100} \times 11 = \frac{11 \times 37}{100}$

Quantity A	Quantity B
26% of 32	8

Answer: (A) Quantity A is greater.

26% is a bit over 25%, which equals $\frac{1}{4}$. So Quantity A is a bit over $\frac{1}{4}$ of 32, or a bit over 8.

At 29 miles per gallon, it takes ____ gallons to make a 319 mile trip.

Answer: 11

$\frac{29 \text{ miles}}{\text{gallon}} \times x \text{ gallons} = 319 \text{ miles}$, so $x = \frac{319}{29} = 11$. Notice that the units "check out": gallons cancel on the left, so both sides of the equation are in terms of miles.

What is the integer equivalent of 2^{11}?

(The answer should not be in exponent form.)

Answer: 2,048

While the calculator could generate the answer, it is faster if you have memorized the powers of 2 up to $1{,}024 = 2^{10}$.

$2^{11} = 2^1\ 2^{10} = 2 \times 1{,}024 = 2{,}048$

Notice that $2^{10} \approx 1{,}000$, so $2^{11} \approx 2{,}000$.

What is the integer equivalent of 2^{20}?

(The answer should not be in exponent form.)

Answer: 1,048,576

While the calculator is helpful, you can reduce the number of buttons you need to push by memorizing the powers of 2 up to $1{,}024 = 2^{10}$.

$2^{20} = (2^{10})^2 = (1{,}024)^2 = 1{,}048{,}576$

Notice that $2^{10} \approx 1{,}000$, so $2^{20} = (2^{10})^2 \approx 1{,}000{,}000$ $(= 1{,}000^2)$.

At a certain university, all of the students major in either engineering or liberal arts (no students major in both). If the ratio of engineering students to liberal arts students is 3/4, what fraction of the students are liberal arts students?

Answer: $\frac{4}{7}$

$\frac{\text{engineering}}{\text{liberal arts}} = \frac{3x}{4x}$, where x is an integer.

The total number of students is $3x + 4x = 7x$.

$$\frac{\text{liberal arts}}{\text{total}} = \frac{4x}{7x} = \frac{4}{7}$$

At a certain university, half of the students major in either engineering or liberal arts, while half of the students major in something else. No students major in more than one field. If the ratio of engineering students to liberal arts students is 3/4, what fraction of the students are engineering students?

Answer: $\frac{3}{14}$

$\frac{\text{engineering}}{\text{liberal arts}} = \frac{3x}{4x}$, where x is an integer.

The number of students majoring in engineering or liberal arts is $3x + 4x = 7x$. These students account for half of the students at the university, so the total number of students is $2(7x) = 14x$.

$$\frac{\text{engineering}}{\text{total}} = \frac{3x}{14x} = \frac{3}{14}$$

Quantity A	Quantity B
The units digit of (8) (3) (7) (5) (9)	0

Answer: (C) The two quantities are equal.

(8) (3) (7) (5) (9) has factors of 5 and 2 (in the 8), so it is a multiple of 10. The units digit is therefore 0.

Quantity A	Quantity B
59% of \$104	\$60

Answer: (A) Quantity A is greater.

Quantity A: 59% of \$104 = 0.59 × \$104 = \$61.36

It can easily be seen that 60% of \$100 would equal \$60, but a smaller percent of a larger dollar amount is worth calculating to be certain.

If you want to try mental math, note that 60% of \$104 is 60% of \$100 plus 60% of \$4, or \$60 + \$2.40 = \$62.40. One percent of \$104 is \$1.04. Subtracting \$1.04 from \$62.40 still leaves you above \$60.

Fill in the blank:

$3^{2^n} = 3^n \times$ _____

Answer: 3^n

$$3^{2n} = 3^n \times blank$$

$$\frac{3^{2n}}{3^n} = blank$$

$$3^{2n-n} = blank$$

$$3^n = blank$$

Fill in the blank:

$$3^{n+2} = 3^n \times \_\_\_\_$$

Answer: 3^2

$$3^{n+2} = 3^n \times blank$$

$$\frac{3^{n+2}}{3^n} = blank$$

$$3^{n+2-n} = blank$$

$$3^2 = blank$$

Last year, the tax revenue for Town A was $120,120. If tax revenue increased by 15% this year, what was tax revenue this year?

Answer: \$138,138

An increase of 15% means that the revenue this year was 1.15 times the revenue last year: $\$120{,}120 \times 1.15 = \$138{,}138$.

If you want to try mental math, treat 15% as 10% plus 5%. 10% of \$120,120 is \$12,012. 5% is half of that, or \$6,006. Add these two number to get \$18,018, and finally add that to \$120,120 to get \$138,138.

The arithmetic mean of Set P is 14 and the standard deviation of Set P is 2.3. If x is a number in Set P that is three standard deviations below the mean, what is the value of x?

Answer: 7.1

Three standard deviations is $3 \times 2.3 = 6.9$. If x is three standard deviations below the mean of 14, then $x = 14 - 6.9 = 7.1$.

To increase a number by 125%, multiply by ____.

Answer: 1.25, or 5/4

$$125\% = \frac{125}{100} = 1.25$$

The fractional equivalent of 1.25 is 5/4.

To increase a number by 10% three times sequentially, multiply by ____.

Answer: $(1.1)^3$ or 1.331

To increase a number by 10%, multiply by 1.1. To do so again, multiply by 1.1 again. To increase by 10% three times sequentially, multiply by $(1.1)(1.1)(1.1)$.

To take a 25% discount, then a 10% discount, multiply by ______.

245

Answer: 0.675, or 27/40

To decrease a number by 25%, multiply by $(1 - 0.25) = 0.75$.

To decrease a number by 10%, multiply by $(1 - 0.10) = 0.9$.

To make these decreases sequentially, multiply by $(0.75)(0.9) = 0.675$.

Notice that the order in which the discounts are taken doesn't matter!

Alternatively, use fractional equivalents:

$0.75 = 3/4 \quad 0.9 = 9/10$

The product is 27/40.

Given $x \neq 0$, simplify: $\frac{x^{-4}}{x^{-5}}$

Answer: x

$$\frac{x^{-4}}{x^{-5}} = x^{(-4)-(-5)} = x^{-4+5} = x^{1} = x$$

Translate into an equation:

A percent of *B* is *C*

Answer: $\frac{A}{100} \times B = C$

"A percent" translates to "$\frac{A}{100}$"

"of B" translates to "$\times B$"

"is C" translates to "$= C$"

Check: If A percent is 5% and B is 40, then 5% of 40 is 2, which is equal to C. Plug these numbers into the equation to make sure it is true:

$\frac{5}{100} \times 40 = \frac{200}{100} = 2$. True.

Translate into an equation:

Julie has 10 more course credits than Mike and Larry combined.

Answer: $J = 10 + M + L$

"Julie has" translates to "$J =$"

"10 more course credits than" translates to "10 +" [Add to whatever noun(s) follow the "more...than."]

"Mike and Larry combined" translates to "$M + L$" [Since Mike and Larry follow the "more...than," the 10 is added to them.]

Check: If Mike has 5 credits and Larry has 8, combined they have 13, and Julie has 10 more than that, or 23. Plug these numbers into the equation:
$23 = 10 + 5 + 8$. True, so the equation is correct.

Translate into an equation:

The average rainfall on Monday, Tuesday, and Wednesday was 3 times the average rainfall on Friday and Saturday.

Answer: $\frac{M+T+W}{3}=3\times\frac{F+S}{2}$

"The average rainfall on Monday, Tuesday, and Wednesday" is $\frac{M+T+W}{3}$

"was" translates to "="

"3 times" translates to "3 × " [Multiply by what follows.]

" the average rainfall on Friday and Saturday" is $\frac{F+S}{2}$

At a certain theater, mezzanine tickets cost \$110 and balcony tickets cost \$20. The 20 tickets purchased by a group had a total cost between \$1,850 and \$2,010. How many balcony tickets did the group buy?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

Answer: (D) 3

Try pairs of numbers:

Mezzanine	Balcony	Total Cost
15	5	15($110) + 5($20) = $1,750
16	4	16($110) + 4($20) = $1,840
17	**3**	**17($110)** + **3($20)** = **$1,930**
18	2	18($110) + 2($20) = $2,020

You can also start by assuming all 20 tickets were mezzanine, for a cost of $2,200, then subtract $90 for each cheaper balcony ticket you swap in ($110 – $20 = $90) until you are in the right range.

What is the average of the set $\{1, 5, 9, 13, 17, 21, 25\}$?

Answer: 13

Notice that the terms in this set are evenly spaced: 5 is 4 greater than 1, 9 is 4 greater than 5, etc.

In an evenly spaced set, the middle term equals the average. There are 7 terms in this set, so the 4^{th} term is the middle term: Average = 13.

Alternatively, add "easy pairs" (e.g., 13 + 17 = 30) and get a total of 91, then divide by 7.

Translate:

Shipping costs for a package are \$5 for the first 5 pounds or less, plus an additional \$0.40 for each additional pound.

Answer: If $x \le 5$, Cost = \$5

If $x > 5$, Cost = \$5 + \$0.40$(x - 5)$

Let's call x the weight of the package, in pounds.

"Shipping costs for a package are \$5 for the first 5 pounds or less," so if $x \le 5$, Cost = \$5.

"plus an additional \$0.40 for each additional pound" over 5 pounds, which is an additional \$0.40$(x - 5)$. So if $x > 5$, Cost = \$5 + \$0.40$(x - 5)$.

The 4 women competing in a race each finished in an average of 22 minutes and 35 seconds. The 3 men competing in the same race each finished in an average of 21 minutes and 45 seconds.

Quantity A	Quantity B
The average race-completion time for all 7 racers.	22 minutes and 8 seconds

Answer: (A) Quantity A is greater.

If the number of men and women racers were equal, the average time for ALL racers would be the simple average of the men's and women's average times, or 22 minutes and 10 seconds (±25 seconds, exactly between each gender's average time).

But this group has slightly more women than men. The women's average time is greater than the simple average, so more women pull the average up. Therefore, the average time for all racers must be greater than 22 minutes and 10 seconds, which is already greater than Quantity B (22 minutes and 8 seconds).

How many different ways can the letters in the word "PLATE" be arranged?

Answer: 120

PLATE has 5 letters, all unique, so there are $5! = 120$ different ways to arrange the letters.

How many different ways can the letters in the word "SPOON" be arranged?

Answer: 60

SPOON has 5 letters, but the letter O appears twice, so there are $\frac{5!}{2!}=\frac{(5)(4)(3)(2)(1)}{(2)(1)}=(5)(4)(3)=60$ different ways to arrange the letters.

How many different ways can the letters in the word "FORK" be arranged?

Answer: 24

FORK has 4 letters, all unique, so there are $4! = 24$ different ways to arrange the letters.

If $(x - 3) = 4$, what is the value of x?

Answer: $x = 7$

$(x - 3) = 4$, implies that $x = 4 + 3 = 7$. This is a linear equation, so there is one solution.

If $\sqrt{(x-3)^2} = 4$, what is the value of x?

Answer: 7 or −1

$\sqrt{(x-3)^2} = 4$ implies that $(x - 3)^2 = 16$, which can happen when $(x - 3) = +4$ or $(x - 3) = -4$. There are two solutions.

If $x - 3 = 4$, then $x = 7$.

If $x - 3 = -4$, then $x = -1$.

Check:

$\sqrt{(7-3)^2} = \sqrt{(4)^2} = \sqrt{16} = 4$ and $\sqrt{(-1-3)^2} = \sqrt{(-4)^2} = \sqrt{16} = 4$.

$\sqrt{(x-3)^2} = |x-3|$, so $x - 3 = 4$ or -4.

$\{-5, -3, -2, -\frac{1}{2}, 0, \frac{1}{4}, \frac{11}{20}, 1, 1, 2, 2, 4, 8, 9, 10\}$

If a 16th term is added to the 15-term set above, the new term would be in Quartile 3 if it were which of the following? Indicate all such values.

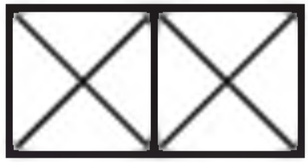
[A] $\frac{1}{3}$ [B] $\frac{5}{4}$ [C] $\frac{8}{3}$ [D] 3 [E] $\frac{7}{2}$ [F] 5

Answer: [B], [C], [D], and [E]

The set is already ordered from least to greatest. Split the 15-term set into quartiles, with 4 terms in Quartiles 1, 2 and 4, leaving space for the 16^{th} term somewhere in Quartile 3:

$$\left\{-5, -3, -2, -\frac{1}{2} \;\middle|\; 0, \frac{1}{4}, \frac{11}{20}, 1 \;\middle|\; 1, \_\_, 2, 2 \;\middle|\; 4, 8, 9, 10\right\}$$

The new term will fall in Quartile 3 if it is between 1 and 4. The answer choices in this range are $\frac{5}{4}$, $\frac{8}{3}$, 3, and $\frac{7}{2}$.



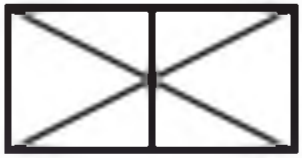
How many different triangles are in the figure above?

Answer: 18

There are 8 small triangles:   etc.

There are 8 medium triangles (comprised of 2 small triangles):   etc.

There are 2 large triangles (comprised of 4 small triangles):  



How many different triangles are in the figure above?

Answer: 12

There are 6 small triangles:   etc.

There are 2 medium triangles (comprised of 2 small triangles):  

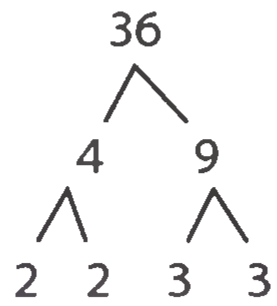
There are 4 large triangles (comprised of 3 small triangles):  
etc.

<u>Quantity A</u>	<u>Quantity B</u>
$\frac{8}{98}$	0.08

Answer: (A) Quantity A is greater.

Resist the urge to reduce $\frac{8}{98}$ to $\frac{4}{49}$. The numbers are easier to compare if we make use of the 8 in both.

Quantity B: $0.08 = \frac{8}{100}$, which has the same numerator but larger denominator than the fraction in Quantity A. Larger denominator means smaller value.

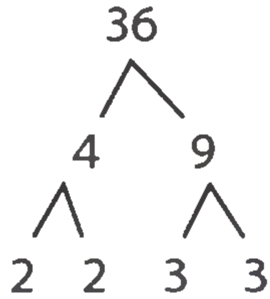


What are the unique prime factors of 36^2?

Answer: 2 and 3

36^2 has the same prime factors as 36. From the factor tree, the prime factors are 2 and 3.

$36^2 = (2^2 3^2)^2 = 2^4 3^4$. There are no new prime factors.



How many factors does 36 have?

Answer: 9

From the factor tree, $36 = 2^2 3^2$. These prime factors can be combined to create all the other factors of 36. These factors can include no, one, or two 2's (3 possibilities) and no, one, or two 3's (3 possibilities). Thus, there are $3 \times 3 = 9$ possible ways to combine the prime factors. Alternatively, list the factors:

	Small	Large	
1	1	36	$2 \times 2 \times 3 \times 3$
2	2	18	$2 \times 3 \times 3$
3	3	12	$2 \times 2 \times 3$
2×2	4	9	3×3
2×3	6	6	(Only count 6 once!)

A factory can produce 190 widgets per metric ton of zinc, and a metric ton of zinc costs $2,060. What is the cost for the zinc used to produce 450 widgets, rounded to the nearest hundred dollars?

Answer: $4,900

Zinc required: 450 widgets/190 widgets per ton = 45/19 metric tons

Cost per metric ton of zinc = $2,060

Total cost = (45/19)($2,060) = $4,878.95. Round up to $4,900.

This is a really tough one for mental math, but we could get close:

$45/19 = 2 + 7/19 \approx 2 + 1/3$
$2 \times \$2{,}060 = \$4{,}120$
$1/3$ of $\$2{,}060 \approx 1/3$ of $\$2{,}100 = \$700$
The sum is $4,820—off by only a little.

A triangle has a perimeter of 24.

What is the maximum area of the triangle?

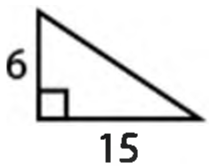
Answer:

To maximize area for a given perimeter, make a regular polygon (one with all sides equal and all angles equal). For a triangle with perimeter 24, this is an equilateral triangle with sides 8. One of these sides will be the base of the triangle. The height of an equilateral triangle is always   times the side length. Here, the height is

A triangle has one side that is 6 inches and another that is 15 inches. What is the maximum area of the triangle?

Answer: 45 inches^2

If two sides of a triangle are given, the area is maximized then those two sides are placed perpendicular to each other. Doing so makes the given sides the base and height:



$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(15)(6) = 45$$

Any other angle between the 6 and the 15 makes the height less than 6.

The sum of five integers is 17. The integer 10 is then included in the set. What is the average of the complete set of six integers?

Answer: 4.5

In order to answer, you don't need to know what the original five integers are individually. Only the sum is required.

The sum of all six integers is just the sum of the first five integers (17), plus the newly included integer 10.

$$\text{Average} = \frac{17+10}{6} = \frac{27}{6} = \frac{9}{2} = 4.5$$

Set S: {11, 18, 45}

If 4 were added to each of the terms in the set above, the standard deviation of the set would ________.

(A) decrease

(B) remain the same

(C) increase

Answer: (B) remain the same

After adding 4 to each of the terms in the set, the average of the terms in new Set S' would be 4 greater than the average of the terms in original Set S. So, if each term increases by 4 and the average increases by 4, the difference between each term and the average would remain the same. Standard deviation is simply a measure of these differences for the whole set, so the standard deviation also would remain the same.

Set S: $\{11, 15, 31\}$

If each of the terms in the set above were multiplied by -3, the standard deviation of the set would ________.

(A) decrease

(B) remain the same

(C) increase

Answer: (C) increase

When each term in the set is multiplied by -3, the average of the new set would become -3 times the original average. The absolute value of the difference between each term and the average of the set would also be $|-3| = 3$ times the original difference.

Standard deviation is simply a measure of these differences for the whole set, so the fact that these differences increase by a factor of 3 means that the standard deviation would increase.

Don't be fooled by the negative sign. The numbers all get farther apart when multiplied by -3.

Set Q: $\{-5, 20, 145\}$

If each of the terms in the set above were multiplied by $\frac{1}{5}$, the standard deviation of the set would ________.

(A) decrease

(B) remain the same

(C) increase

Answer: (A) decrease

When each term in the set is multiplied by $\frac{1}{5}$, the average of the new set would become $\frac{1}{5}$ times the original average. The positive difference between each term and the average of the set would also be $\frac{1}{5}$ of the original difference.

Standard deviation is simply a measure of these differences for the whole set, so the fact that these differences decrease by a factor of 5 means that the standard deviation would decrease.

Set R: {1, 2, 3, 4, 5}

If each of the terms in the set above were multiplied by 2 then had 2 subtracted from the it, the standard deviation of the set would ________.

(A) decrease

(B) remain the same

(C) increase

Answer: (C) increase

The average of the terms in Set R is 3 (the middle term in the evenly spaced set). The positive differences between each term and the average are 2, 1, 0, 1, and 2, respectively.

The new Set R' would be {0, 2, 4, 6, 8}. The average would be 4, and the positive differences between each term and the average would be 4, 2, 0, 2, and 4, respectively.

Standard deviation is simply a measure of these differences for the whole set, so the fact that these differences would double means that the standard deviation would increase. We can ignore the subtraction and just pay attention to the multiplication by 2.

Set R: $\{-50, -49, -48\}$

If each of the terms in the set above had 50 subtracted from it, then was doubled, the standard deviation of the set would ________.

(A) decrease

(B) remain the same

(C) increase

Answer: (C) increase

The average of the terms in Set Z is –49 (the middle term in the evenly spaced set). The positive differences between each term and the average are 1, 0, and 1, respectively.

The new Set Z′ would be {–200, –198, –196}. The average would be –198, and the positive differences between each term and the average would be 2, 0, and 2, respectively.

Standard deviation is simply a measure of these differences for the whole set, so the fact that these differences would double means that the standard deviation would increase. We can ignore the subtraction and just pay attention to the doubling.

In a set of 1,000 consecutive multiples of 3, what is $P_{62} - P_{12}$?
(Remember, P_{62} is the 62^{nd} percentile of the set, where each percentile is comprised of $\frac{1{,}000}{100} = 10$ terms.)

Answer: 1,500

If each percentile is composed of 10 terms, and P_{62} and P_{12} are 50 percentiles apart ($62 - 12 = 50$), then P_{62} and P_{12} are (10 terms/percentile)(50 percentiles) = 500 terms apart.

The terms are consecutive multiples of 3. Terms that are 1 apart (i.e. adjacent terms) differ by 3. Terms that are 2 terms apart differ by $(2)(3) = 6$. Terms that are 500 terms apart differ by $(500)(3) = 1{,}500$.

If two fair 6-sided dice are rolled together, what is the probability of rolling numbers that have an even product?

Answer: $\frac{3}{4}$

An even product results from (E)(O) or (E)(E) or (O)(E). The only way to get an odd product is (O)(O). Thus, out of these 4 basic cases, each of which is equally probable, 3 case result in an even product.

Alternatively, think of all the possible number pairs. There are 6 number possibilities for the first die, and 6 for the other, for a total of (6)(6) = 36 possible pairings. How many of these are (O)(O), the “failing” cases? There are 3 odd possibilities for the first die, and 3 for the other, for a total of (3)(3) = 9 possible (O)(O) pairings. Thus, 36 – 9 = 27 pairings have an even product, and $\frac{27}{36} = \frac{3}{4}$.

If two fair 6-sided dice are rolled together, what is the probability of rolling numbers that have an even sum?

Answer: $\frac{1}{2}$

An even sum results from (E + E) or (O + O). An odd sum results from (E + O) or (O + E). There are no other possibilities. Thus, out of these 4 basic cases, 2 result in an even sum. $\frac{2}{4}=\frac{1}{2}$

If two fair 6-sided dice are rolled together, what is the probability of rolling numbers that have a product of 12?

Answer: $\frac{1}{9}$

Using the numbers 1 through 6, what pairs have a product of 12? (2×6) or (3×4) or (4×3) or (6×2). Note that the order of the numbers doesn't matter for the product (i.e. $12 = 3 \times 4 = 4 \times 3$), but the fact that the 3 could occur on *either* die, while the 4 is on the other, doubles the chance of pairing 3 with 4. The same is true for 2 and 6.

In all, there are 6 number possibilities for the first die, and 6 for the other, for a total of $(6)(6) = 36$ possible pairings. As shown above, 4 of these pairings have a product of 12, so the probability is $\frac{4}{36} = \frac{1}{9}$.

Quantity A

The area of a regular hexagon with each side equal to $\frac{x}{6}$

Quantity B

The area of a square with each side equal to $\frac{x}{4}$

Answer: (A) Quantity A is greater

A hexagon has 6 sides, so the perimeter of the hexagon is x. A square has 4 sides, so the perimeter of the square is also x.

For a given perimeter, area is maximized by making a polygon as regular as possible (all sides the same, all angles the same) and as multi-sided as possible. In the extreme, consider a regular polygon with 200 sides—it is almost a circle, which is the shape with maximum area for a given perimeter. Thus, for perimeter x, a regular hexagon has a greater area than a regular square.

A fair coin is flipped three times. What is the probability of flipping tails exactly once?

Answer: $\frac{3}{8}$

On each flip, either a head (H) or tails (T) will result. There are 2 possible outcomes of 1 flip. Thus, there are $2 \times 2 \times 2 = 8$ ways the series of three flips could go.

There are 3 ways that tails could be flipped "exactly once": THH or HTH or HHT. Therefore, the probability is $\frac{3}{8}$.

Factor: $ab + 2b - 3a - 6$

Answer: $(a + 2)(b - 3)$

Group terms with similarities, and then factor out the shared element:

$ab + 2b - 3a - 6$

$(ab - 3a) + (2b - 6)$

$a(b - 3) + 2(b - 3)$

$(a + 2)(b - 3)$

To check, FOIL the factored expression back to its original form.

Factor: $a^2 + ax + \frac{x^2}{4}$

Answer: $\left(a+\frac{x}{2}\right)^2$

$$a^2+ax+\frac{x^2}{4}=a^2+ax+\left(\frac{x}{2}\right)^2$$
$$=\left(a+\frac{x}{2}\right)\left(a+\frac{x}{2}\right)$$

Check: When FOILing, the Outer and Inner terms will each be $\frac{ax}{2}$, and there are two of them, so they will sum to *ax*, the middle term in the original expression. The First and Last terms are just the squares of the respective terms in the factored form.

The "length" of a positive integer is the number of non-unique prime factors it has. For example, $60 = (2)(2)(3)(5)$ has a length of 4. What is the maximum length of the numbers between 1,500 and 2,000, inclusive?

(A) 6

(B) 7

(C) 8

(D) 9

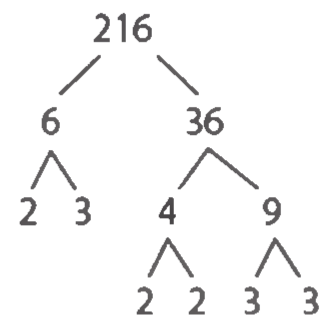
(E) 10

Answer: (E) 10

To maximize the "length" of a positive integer while also limiting its value, use as many factors of 2 (the smallest prime) as possible, and avoid using large primes, such as 47 or 89. The powers of 2 near our number range are: $2^9 = 512$, $2^{10} = 1{,}024$, and $2^{11} = 2{,}048$.

Since $2{,}048 > 2{,}000$, length of 11 is not possible. What about length of 10? If all 10 primes are 2, the resulting number is too small (1,024), but try using a 3, the next smallest prime. After a little trial and error: $3^1(2^9) = 3(512) = 1{,}536$. This is in the range, and has a length of $1 + 9 = 10$.

Solve for x: $x^2 + 6x - 216 = 0$



Answer: $x = 12$ or -18

The factors of 216 are:

Small	Large
1	216
2	108
3	72
4	54

Small	Large
6	36
8	27
9	24
12	18

Since 216 is negative in the quadratic, the factor numbers will have opposite signs. Look for the factor pair that *differs* by 6, in order to create the $+6x$ middle term of the quadratic: $(x - 12)(x + 18) = 0$

What is the greatest integer less than 65 that has at least 3 unique prime factors?

Answer: 60

Consider the numbers less than 65, starting with the greatest such number. Factor and count unique prime factors.

$64 = 2^6$	1 unique prime factor: 2
$63 = (3)(3)(7)$	2 unique prime factors: 3, 7
$62 = (2)(31)$	2 unique prime factors: 2, 31
61 is prime!	1 unique prime factor: 61
$60 = (2)(2)(3)(5)$	3 unique prime factors: 2, 3, and 5

What are the first 15 prime numbers?

Answer: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

Quantity A	Quantity B
$\dfrac{1.75}{\dfrac{18}{4}-\dfrac{11}{25}}$	$\dfrac{1}{2}$

Answer: (B) Quantity B is greater.

Since Quantity A mixes decimals and fractions, you could solve using either form. Because Quantity A is compared to 1/2, the question might be rephrased as "How does the denominator compare to 2(1.75) = 3.50?"

Quantity A:

$$\frac{1.75}{\frac{18}{4}-\frac{11}{25}} = \frac{1.75}{4.5-\frac{11}{25}} = \frac{1.75}{4.5-\frac{44}{100}} = \frac{1.75}{4.5-0.44} = \frac{1.75}{4.06}$$

4.06 > 3.50, so Quantity A is less than 1/2.

What could be the value of x if...

...$x^2 = 36$?

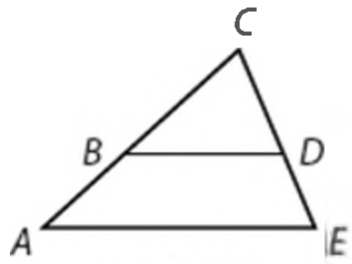
$$\ldots\ x = \sqrt{36}\,?$$

Answer: 6 or -6 (top question), but only $+6$ (bottom question)

$x^2 = 36$ has two solutions: $x = 6$ and $x = -6$. Plug back in to see why.

$6^2 = 36$ and similarly $(-6)^2 = 36$.

$x = \sqrt{36}$ has only one solution: $x = 6$. Notice that $-6 \neq \sqrt{36}$.



$BD \parallel AE$

In the figure above, $\frac{CD}{CE} = \frac{2}{3}$, and the length of AE is 7.5. What is the length of BD?

Answer: 5

Line segment *BD* drawn within triangle *ACE* parallel to side *AE* creates a **Similar Triangle** *BCD*. Similar triangles have the same three angle measures, and the ratio of corresponding sides of the two triangles is constant.

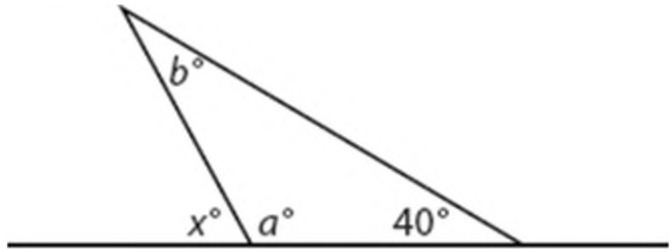
In equation form, this means that the given side ratio is equivalent to the ratio of *BD* to *AE*: $\frac{2}{3}=\frac{CD}{CE}=\frac{BD}{AE}$

So,

$$\frac{2}{3}=\frac{BD}{7.5}$$

$$\frac{2}{3}(7.5)=BD$$

$$BD=5$$



Based on the figure above, what is $x - b$?

Answer: 40

Angles that form a line sum to 180c, so $x + a = 180$

The sum of the angles in a triangle is 180°, so $a + b + 40 = 180$.

The two expressions that equal 180 can be set equal to each other and solved.

$x + a = a + b + 40$

$x = b + 40$

$x - b = 40$

Alternatively, an exterior angle of a triangle is equal to the sum of the two opposite interior angles of the triangle. This means that $x = b + 40$, or $x - b = 40$.

Two right circular cylindrical glasses hold the same volume of tea when filled to the brim. The glass with diameter 5 is how many times the height of the glass with radius 5?

Answer: 4

In order to hold the same volume of tea, one glass is tall and narrow (diameter 5, so radius 5/2), the other glass is short and wide (diameter 10, since radius 5).

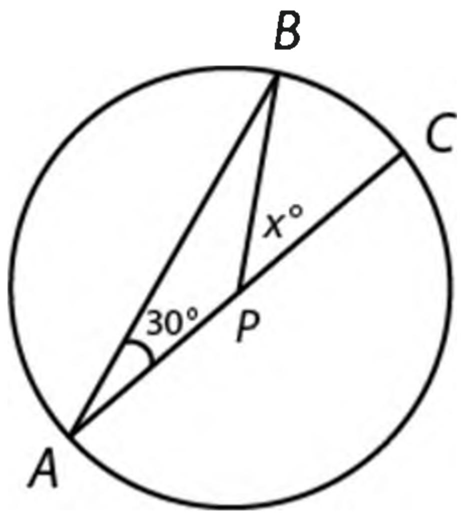
Tall cylinder volume $= \pi\left(\frac{5}{2}\right)^2 h_{\text{tall}}$

Short cylinder volume $= \pi 5^2 h_{\text{short}}$

The volume of tea is equal:

$$\pi\left(\frac{5}{2}\right)^2 h_{\text{tall}} = \pi 5^2 h_{\text{short}} \qquad \left(\frac{25}{4}\right) h_{\text{tall}} = 25 h_{\text{short}} \qquad \frac{h_{\text{tall}}}{h_{\text{short}}} = 25 \times \frac{4}{25} = 4$$

Intuitively, the glass with radius 5 has twice the radius, so the area of its base is four (= 2^2) times as big. So the other glass must be four times as tall, to compensate.



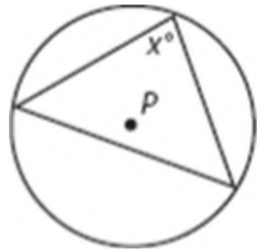
If Point P is the center of the circle in the figure above, what is x?

Answer: 60

There are two ways to determine the answer.

(1) Since *AP* and *PB* are both radii of the circle, triangle *ABP* is an isosceles triangle, so $\angle ABP$ is also 30. Thus, the remaining angle in the triangle, $\angle APB$, is $180 - 30 - 30 = 120$. Since x and $\angle APB$ form a straight line, $x = 180 - \angle APB = 180 - 120 = 60$.

(2) Recognize that for minor arc *BC*, 30° labels the inscribed angle, and $x°$ labels the central angle. For a given arc, the inscribed angle is half of the central angle, or the central angle is twice the inscribed angle. $2(30) = 60$.



In the figure above, point P is the center of the circle, and the radius of the circle is 1.

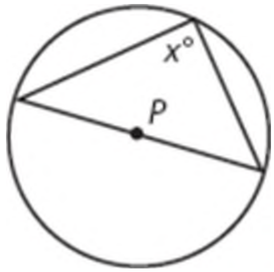
What is x if point P does NOT lie on any side of the triangle and the area of the triangle is maximized?

Answer: 60

Because the center of the circle does NOT lie on any side of the triangle, x cannot be 90. However, x could be any *other* value greater than 0 and less than 180.

But given the circular limits for this triangle, area is maximized by making the triangle as regular as possible (all sides the same, all angles the same). In other words, an equilateral triangle maximizes the triangle area. Thus, $x = 60$, as do the other two angles in the triangle.

The formal proof that the equilateral triangle maximizes the area in this case is not trivial, but you're not responsible for that proof. Just remember that regular polygons (such as the equilateral triangle) maximize the area under constraints (being inside a circle, or having a constant perimeter, etc.).



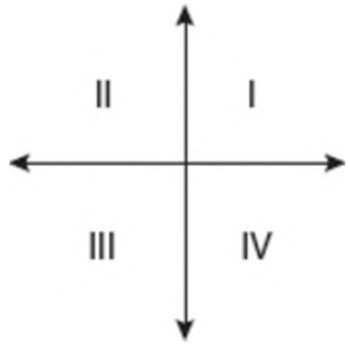
In the figure above, point P is the center of the circle and it lies on one side of the triangle.

What is the maximum area of the triangle if the radius of the circle is 1?

Answer: 1

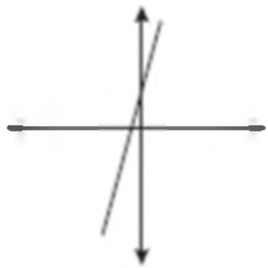
Because the center of the circle lies on one side of the triangle, the opposite angle of the triangle is a right angle: $x = 90$. The hypotenuse of this right triangle is 2(radius) = 2. To maximize the area of the triangle, make the two perpendicular sides of the right triangle the same length, i.e. make a 45–45 –90 triangle. This makes the height as large as possible. For a hypotenuse of 2, the other side lengths would be $\frac{2}{\sqrt{2}}$.

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}\left(\frac{2}{\sqrt{2}}\right)\left(\frac{2}{\sqrt{2}}\right) = \frac{4}{4} = 1$$



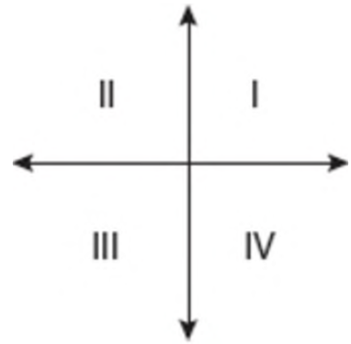
Which quadrants will $y = 4x + \frac{5}{3}$ go through?

Answer: I, II, and III



The *y*-intercept of this line is $\frac{5}{3}$, which is positive. The line crosses the *y*-axis between quadrants I and II.

The slope of the line is +4, which means the line slopes up into quadrant I and down into quadrant II, passing also down into quadrant III.

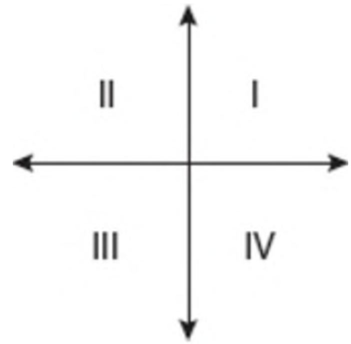


Which quadrants will $y = -3x + 7$ go through?

Answer: I, II, and IV

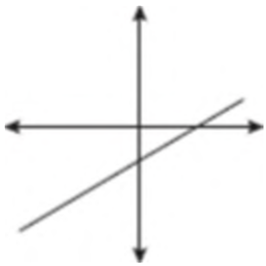
The y-intercept of this line is 7, which is positive. The line crosses the y-axis between quadrants I and II.

The slope of the line is -3, which means the line slopes up into quadrant II and down into quadrant I, passing also down into quadrant IV.



Which quadrants will $y = \frac{1}{2}x - 13$ go through?

Answer: I, III, and IV



The y-intercept of this line is –13, which is negative. The line crosses the y-axis between quadrants III and IV.

The slope of the line is +1/2, which means the line slopes up from left to right: from quadrant III into quadrant IV, and finally up into quadrant I.

Quantity A	Quantity B
The maximum possible circumference of a spherical ball that would fit inside a 10 by 11 by 14 rectangular box.	14π

Answer: (B) Quantity B is greater.

The ball cannot have a diameter greater than 10, the smallest dimension of the box. The maximum circumference of the ball is like the "equator" of the ball, where $d = 10$. Circumference $= \pi d = 10\pi$.

If a right circular cylindrical can has radius 3.75 and height 11, what is the surface area of the curved surface of the can (that is, excluding the flat bases)?

298

Answer: 82.5π

If the curved surface of the can were unrolled, it would be a rectangle with height = height of the original can, and width = circumference of the flat circular ends of the can.

Rectangle width = can circumference = $2\pi r = 2\pi(3.75) = (7.5)\pi$.

Rectangle area = $wh = (7.5\pi)(11) = 82.5\pi$

x, y, and z are integers. If $\frac{x}{y}$ is a negative integer, $xz > 0$, and y is even, which of the following statements could be true?

Select all such statements.

[A] x is even

[B] x is odd

[C] x is positive

[D] z is negative

Answer: [A], [C], and [D].

$\frac{x}{y}$ is a negative integer, so *x* and *y* have opposite signs.

$xz > 0$, so *x* and *z* have the same sign.

x	*y*	*z*
+	-	+
-	+	-

Both choice [C] and [D] could be true.

Since $\frac{x}{y}$ = an integer, then $x = y(\text{integer}) = \text{even}(\text{integer}) = \text{even}$. Choice [A] must be true. Because *x* cannot be odd, choice [B] is eliminated.

What is the sum of the 30 smallest even positive integers?

300

Answer: 930

The first even integer is 2, the second is 2(2) = 4, and the third is 3(2) = 6, so the thirtieth even integer is 30(2) = 60. What is the sum of 2, 4, 6,..., 58, and 60?

$$\text{Sum} = \text{Number of Terms}\left(\frac{\text{First} + \text{Last}}{2}\right) = 30\left(\frac{2+60}{2}\right) = 30(31) = 930$$

If the sum of the consecutive integers from x to $(x + 6)$, inclusive, is 56, what is x?

301

Answer: 5

There are 7 integers in the sum, so the average integer is $\frac{56}{7}=8$. In a consecutive set, the average is also the middle term. Thus, the 4^{th} integer in this list is 8: {5, 6, 7, 8, 9, 10, 11}. *x* is 5.

Alternatively,

$$56=x+(x+1)+(x+2)+(x+3)+(x+4)+(x+5)+(x+6)$$
$$56=7x+(1+2+3+4+5+6)$$
$$56=7x+21$$
$$35=7x$$
$$x=5$$

What is the average of 11.5, 12.25, 13, 13.75, 14.5, 15.25, and 16?

Answer: 13.75

Give the calculator a rest! The terms in this set are equally spaced 0.75 apart. In an evenly spaced set, the average of the set equals the middle term (if an odd number of terms) or the average of the two middle terms (if an even number of terms). This set has 7 terms, so the middle term is the 4^{th}, which is 13.75.

Quantity A	**Quantity B**
The average of $-2x$, $-x$, 0, x, $2x$, and $3x$.	0

Answer: (D) The relationship cannot be determined from the information given.

In Quantity A, the 6 terms are evenly spaced x apart. The average of the set is the average of the two middle terms (since there are an even number of terms), which is $\frac{0+x}{2}=\frac{x}{2}$. Be careful! It is tempting to think this list is ordered from low to high, negative to positive, but that is only true if x is positive. If x is negative, the list is ordered high to low, positive to negative. Either way, the average is $\frac{x}{2}$, but this could be greater or less than 0. In fact, x could even be 0, making all the terms and the average in Quantity A equal 0.

Raffle tickets with consecutive integers 1,014 to 2,345, inclusive, were sold. Each ticket has exactly one unique integer on it. How many tickets were sold?

Answer: 1,332

Subtract the low number from the high number and "add one before you are done."

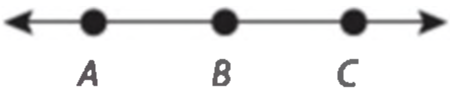
$$\begin{array}{r} 2,345 \\ -1,014 \\ \hline 1,331 \\ +1 \\ \hline 1,332 \end{array}$$

A, B, and C lie on a number line such that B is the midpoint between A and C.

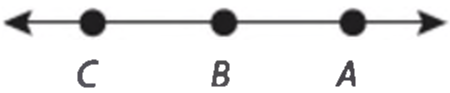
Quantity A	Quantity B
A	B

Answer: (D) The relationship cannot be determined from the information given.

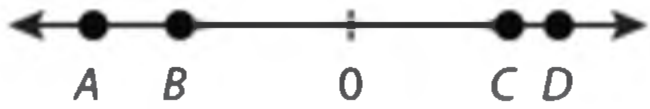
The number line could look like this:



Or like this:



Either A or B could be the greater number.



Note: Figure not drawn to scale

Quantity A	Quantity B
$A \times B$	$C \times D$

Answer: (D) The relationship cannot be determined from the information given.

From the number line, it is clear that A and B are both negative and C and D are both positive. Thus, both quantities are positive, but no scale is provided, so nothing is known about the relative values.

One possibility is that $A = -3$, $B = -2$, $C = 4$, $D = 5$. Since $(-3)(-2) = +6$ and $(4)(5) = +20$, Quantity B is bigger.

Another possibility is that $A = -9$, $B = -7$, $C = 7$, $D = 8$. Since $(-9)(-7) = +63$ and $(7)(8) = +56$, Quantity A is bigger.

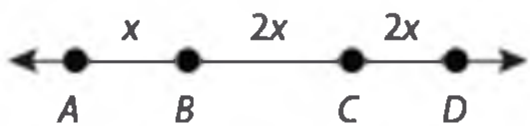
Note: Figure not drawn to scale

On the number line above, B is twice as far from C as from A, and D is twice as far from B as from C. What is the ratio of length $\overline{AD}$ to length $\overline{BC}$?

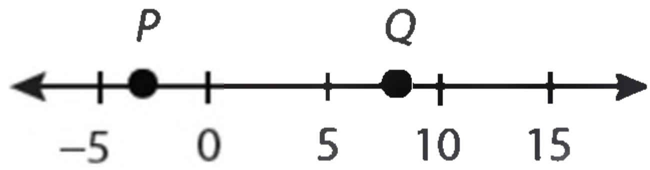
Answer: $\frac{5}{2}$

"*B* is twice as far from *C* as from *A*": Label the distance between *B* and *A* as *x*. The distance between *B* and *C* is twice that, or 2*x*.

"*D* is twice as far from *B* as from *C*": If the distance between *C* and *D* is 2*x*, then *D* is $2x + 2x = 4x$ from *B*, which is "twice as far...as from *C*."



$$\frac{\overline{AD}}{\overline{BC}} = \frac{x+2x+2x}{2x} = \frac{5x}{2x} = \frac{5}{2}$$



Quantity A

Quantity B

$|P|$

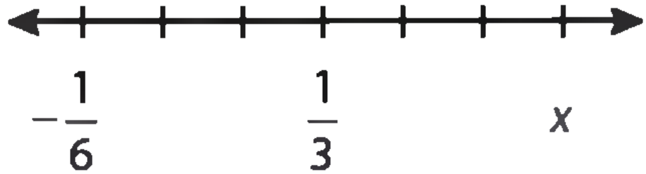
Q

Answer: (B) Quantity B is greater.

P is between -5 and 0, so $|P|$ is between 0 and 5.

Q is between 5 and 10.

Thus, $Q > 5 > |P|$.



If the tick marks on the number line above are evenly spaced, what is the value of x?

Answer: $\frac{5}{6}$

$\frac{1}{3}$ is three tick marks to the right of $-\frac{1}{6}$. In other words, 3 intervals equals

$\frac{1}{3}-\left(-\frac{1}{6}\right)=\frac{2}{6}+\frac{1}{6}=\frac{3}{6}$. Each interval is $-\frac{1}{6}$, then.

x is three tick marks to the right of $\frac{1}{3}$, so x is $\frac{3}{6}$ greater than $\frac{1}{3}$.

$$x=\frac{1}{3}+\frac{3}{6}=\frac{2}{6}+\frac{3}{6}=\frac{5}{6}.$$

Based on the number line above, which of the following must be true? Select <u>all</u> that apply.

[A] $xy < 0$

[B] $zy < xy$

[C] $z - x > x - y$

[D] $z - y > z - x$

Answer: [A], [B] and [D]

Note that only one number, 0, is labeled on the number line. From their relative positions, x and z are positive while y is negative. It is also true that $z > x$. Beyond that, however, the values are not known, and don't assume the figure is drawn to scale.

[A] TRUE: x is positive and y is negative, so $xy < 0$

[B] TRUE: $zy < xy$ divided by y becomes $z > x$ (flip the inequality sign because y is negative). It is true that $z > x$.

[C] UNCERTAIN: Both $(z - x)$ and $(x - y)$ are positive, but without known values, either could be greater.

[D] TRUE: Subtract z from both sides to get $-y > -x$. Divide by -1 (don't forget to flip the inequality sign!) to get $y < x$. This must be true because y is negative and x is positive. Alternatively, $(z - y)$ is the distance between y and z on the line, which is clearly greater than the distance between x and z, $(z - x)$.

Simplify: $\frac{\sqrt{13}\times\sqrt{10}}{\sqrt{65}}$

Answer: $\sqrt{2}$

$$\frac{\sqrt{13}\times\sqrt{10}}{\sqrt{65}}=\sqrt{\frac{13\times10}{65}}=\sqrt{\frac{130}{65}}=\sqrt{2}$$

Simplify: $\frac{\sqrt{84}}{\sqrt{7}}$

Answer: $2\sqrt{3}$

$$\frac{\sqrt{84}}{\sqrt{7}} = \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

Simplify: $\sqrt{45} \times \sqrt{125}$

Answer: 75

$$\begin{aligned}\sqrt{45}\times\sqrt{125} &= \sqrt{45\times125}\\ &= \sqrt{(9\times5)\times(5\times25)}\\ &= \sqrt{9\times25\times25}\\ &= 3\times5\times5\\ &= 75\end{aligned}$$

What is the length of the diagonal of a square with side length $8\sqrt{2}$?

Answer: 16

The diagonal of a square is always $\sqrt{2}$ times the side length. For this square, the diagonal is $\sqrt{2} \times 8\sqrt{2} = 8 \times 2 = 16$.

The diagonal of a square is 10. What is the area of the square?

315

Answer: 50

The diagonal of a square is always $\sqrt{2}$ times the side length. Conversely, the side length is always the diagonal divided by $\sqrt{2}$. For this square, the side length is $\frac{10}{\sqrt{2}}$.

Area of a square is the side length squared: area $= \left(\frac{10}{\sqrt{2}}\right)^2 = \frac{10^2}{(\sqrt{2})^2} = \frac{100}{2} = 50$

The area of a rectangle is 60 and the diagonal length is 13. What is the perimeter of the rectangle?

Answer: 34

When the diagonal of a rectangle is 13, you should at least test the possibility that the diagonal creates two 5–12–13 right triangles. If so, the rectangle area would be $(5)(12) = 60$, which is exactly what this question specifies. Thus, perimeter $= 2(L + w) = 2(12 + 5) = 34$.

If two sides of a triangle are each 8 inches and the third side is x inches, what are possible values for x?

Answer: $0 < x < 16$

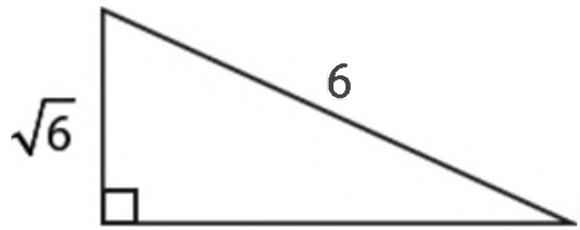
The sum of any two side lengths of a triangle will always be greater than the third side. The sum of 8 and 8 is 16, which must be greater than third side x. The sum of x and either side 8 must be greater than the other side 8: $x + 8 > 8$, or $x > 0$.

Quantity A | **Quantity B**

Quantity A	Quantity B
The area of a triangle with side lengths $\sqrt{3}$, 3, and $2\sqrt{3}$	3

Answer: (B) Quantity B is greater.

The triangle side lengths in Quantity A are special: $\sqrt{3}$, 3, and $2\sqrt{3}$ can be written as $\sqrt{3}(1)$, $\sqrt{3}\left(\sqrt{3}\right)$, and $\sqrt{3}(2)$. In other words, the side lengths are in the ratio $\left(1:\sqrt{3}:2\right)$, which you should recognize as the ratio of the side lengths in a 30–60–90 triangle. Multiplying by $\sqrt{3}$ doesn't change any angles;s it just increases the size of the triangle. Thus, the base and height of this right triangle are $\sqrt{3}$ and 3, so the area $= \frac{1}{2}bh = \frac{1}{2} \times 3 \times \sqrt{3} = \frac{3\sqrt{3}}{2} \approx 2.59$.



Note: Figure not drawn to scale

What is the area of the triangle above?

Answer: $3\sqrt{5}$

The height and hypotenuse of the right triangle are known. Use Pythagorean Theorem to determine the base.

$$a^2 + b^2 = c^2$$

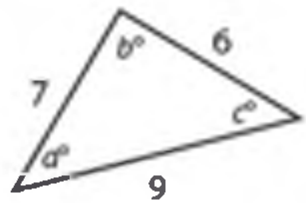
$$\text{base}^2 + \left(\sqrt{6}\right)^2 = 6^2$$

$$\text{base}^2 + 6 = 36$$

$$\text{base} = \sqrt{30}$$

The area of the triangle is

$$\frac{1}{2}bh = \frac{1}{2}\sqrt{30}\sqrt{6} = \frac{1}{2}\sqrt{5}\sqrt{6}\sqrt{6} = \frac{6\sqrt{5}}{2} = 3\sqrt{5}$$



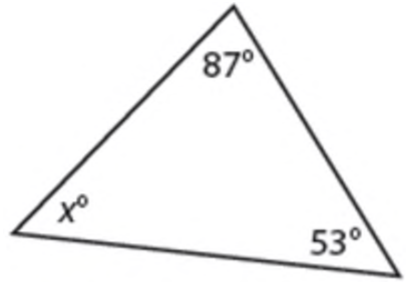
Note: Figure not drawn to scale

Quantity A	Quantity B
a	b

Answer: (B) Quantity B is greater.

The longest side of a triangle is opposite the largest angle, and the smallest side is opposite the smallest angle.

6 is the shortest side of the triangle, so $a°$ is the smallest angle of this triangle. 9 is the longest side of the triangle, so $b°$ is the largest angle of this triangle. Thus, $b > a$.



Note: Figure not drawn to scale

In the figure above, what is x?

Answer: 40

The sum of the angles in a triangle is 180:

$$180 = x + 87 + 53$$

$$180 = x + 140$$

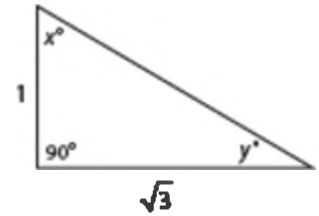
$$40 = x$$

In a triangle with angles $45°$, $45°$, and $90°$, a side length measures $5\sqrt{2}$. What could the other side lengths be?

Answer: 5, $5\sqrt{2}$, and/or 10

In a 45–45–90 triangle, the ratio of the side lengths is $x : x : x\sqrt{2}$.

The known side could be the hypotenuse, in which case the sides are $5 : 5 : 5\sqrt{2}$. Or, the known side could be one of the perpendicular legs, in which case the hypotenuse is $5\sqrt{2}\times\sqrt{2}=5\times2=10$, and the side ratio is $5\sqrt{2} : 5\sqrt{2} : 10$.



Note: Figure not drawn to scale

Quantity A	Quantity B
$\frac{x}{y}$	2

Answer: (C) The two quantities are equal.

This is a right triangle, so by Pythagorean Theorem, the hypotenuse is $\sqrt{a^2+b^2}=\sqrt{1^2+\left(\sqrt{3}\right)^2}=\sqrt{1+3}=2$. The sides of this triangle are in the ratio 1 : $\sqrt{3}$: 2, which you should recognize as a property of a 30–60–90 triangle. The shortest side, 1, is across from the smallest angle: $y = 30$. The middle side, $\sqrt{3}$, is across from the middle angle: $x = 60$.

Therefore, $\frac{x}{y}=\frac{60}{30}=2$.

$$x = 1{,}234{,}567.89$$

Quantity A	Quantity B
The digit in the hundredths place of x	The digit in the hundredths place of x

Answer: (A) Quantity A is greater.

Quantity A: The hundred*ths* place of 1,234,567.8**9** is 9.

Quantity B: The ten thousands place of 1,2**3**4,567.89 is 3.

$9 > 3$

Which of the following equals 48,290?

Select all that are equal.

[A] 482.9×10^2

[B] 48.29×10^{-3}

[C] 0.4829×10^4

[D] $482,900 \times 10^{-1}$

[E] 0.04829×10^6

Answer: [A], [D], and [E]

What is equal to 48,290?

[A] EQUAL. $482.9 \times 10^2 = 48,290$

[B] NOT. $48.29 \times 10^{-3} = 0.04829$

[C] NOT. $0.4829 \times 10^4 = 4,829$

[D] EQUAL. $482,900 \times 10^{-1} = 48,290$

[E] EQUAL. $0.04829 \times 10^6 = 48,290$

What is $\frac{271.39 \times 10^{4}}{0.29818 \times 10^{4}}$, rounded to the nearest integer?

326

Answer: 9

$$\frac{271.39\times10^{4}}{0.29818\times10^{6}}=\frac{2,713,900}{298,180}\approx\frac{2,700,000}{300,000}=\frac{27}{3}=9$$

or

$$\frac{271.39\times10^{4}}{0.29818\times10^{6}}=\frac{271.39}{0.29818\times10^{2}}=\frac{271.39}{29.818}\approx\frac{270}{30}=9$$

0.00021 is what percent of 0.007?

Answer: 3%

Normally, for an "*X* is what percent of *Y*?" question, you would do $\left(\frac{X}{Y}\times 100\right)\%$ in your calculator. While you could just plug these numbers into the calculator and divide, the risk of misplacing the decimal is high, so a secondary "human calculator" check is a good idea.

Shift the decimal in both top and bottom by 5 places before dividing, eliminating decimals altogether.

$$\left(\frac{0.00021}{0.007}\times 100\right)\% = \left(\frac{21}{700}\times 100\right)\% = \left(\frac{21}{7}\right)\% = 3\%$$

What is $0.000002 \times 50,000$?

Answer: 0.1

While you could just plug into the calculator and multiply, the risk of misplacing the decimal means that a secondary "human calculator" check is a good idea.

"Trade" decimal places. Move the decimal 5 to the right in the first number and 5 to the left in the second number, getting:

0.2×0.5

Half of 0.2 is 0.1.

Quantity A	Quantity B
The ten thousandths digit of $\frac{31}{99}$	The hundred thousandths digit of $\frac{11}{20}$

Answer: (A) Quantity A is greater.

$\frac{11}{20}$ is a "terminating" decimal, as there are only prime factors of 2 and 5 in the denominator: 20 = (2)(2)(5). Where does it terminate? $\frac{11}{20} = 0.55$, so every digit past the hundredths place equals 0. Thus, Quantity B is 0.

$\frac{31}{99}$ is a repeating decimal, as there are prime factors other than 2 or 5 in the denominator: 99 = (3)(3)(11). Determine the repeating pattern: $\frac{31}{99} = 0.313131... = 0.\overline{31}$. The hundred thousandths digit is the 5^{th} digit after the decimal, so Quantity A is 3. (Either digit after the decimal would have been greater than 0, actually.)

Change the improper fraction $\frac{89}{3}$ to a mixed number.

Answer: $29\frac{2}{3}$

$\frac{89}{3}$ is $\frac{1}{3}$ short of $\frac{90}{3}=30$. Thus, $\frac{89}{3}$ as a mixed number is $30-\frac{1}{3}=29\frac{2}{3}$.

Alternatively, divide 3 into 89. It goes in evenly 29 times. $(29)(3) = 87$, so $89 - 87 =$ 2 thirds are left over.

Simplify: $\dfrac{4/5}{9/2}$

331

Answer: $\frac{8}{45}$

$$\frac{4/5}{9/2}=\frac{4}{5}\times\frac{2}{9}=\frac{4\times 2}{5\times 9}=\frac{8}{45}$$

Simplify: $\dfrac{7/3}{11/12}$

Answer: $\frac{28}{11}$ or $2\frac{6}{11}$

$$\frac{7/3}{11/12}=\frac{7/3\times12}{11/12\times12}=\frac{7\times4}{11}=\frac{28}{11}$$

Quantity A	Quantity B
$\frac{13}{21}$	$\frac{22}{32}$

Answer: (B) Quantity B is greater.

Both quantities are close to a benchmark value of $\frac{2}{3}$.

Quantity A: $\frac{13}{21}$ is slightly less than $\frac{14}{21}=\frac{2}{3}$, as the numerator is smaller.

Quantity B: $\frac{22}{32}$ is slightly greater than $\frac{22}{33}=\frac{2}{3}$, as the denominator is smaller.

Quantity B > $\frac{2}{3}$ > Quantity A.

Simplify: $\frac{4}{2+16}$

Answer: $\frac{2}{9}$

Don't split the denominator! You must add the numbers in the denominator *before* dividing. Parentheses can serve as a visual reminder, as Parentheses come first in the PEMDAS order of operations.

$$\frac{4}{2+16}=\frac{4}{(2+16)}=\frac{4}{18}=\frac{2}{9}$$

Given $x \neq 2$, simplify: $\frac{4-2x}{5x-10}$

Answer: $-\frac{2}{5}$

$$\frac{4-2x}{5x-10}=\frac{-2x+4}{5x-10}=\frac{-2(x-2)}{5(x-2)}=-\frac{2}{5}$$

$\frac{3}{8}$ of the fish in a certain tank are clownfish. $\frac{2}{5}$ of the other fish were first placed in the tank yesterday. Placing those fish in the tank increased the total number of fish in the tank by what fraction?

Answer: $\frac{1}{3}$

(8)(5) = 40, which is a Smart Number for the total number of fish in the tank, as it is divisible by every denominator in the constraints. If there are 40 fish in the tank, $\frac{3}{8}(40)=15$ of the fish are clownfish. There are 40 − 15 = 25 fish of <u>other</u> types. If $\frac{2}{5}$ of these 25 others were placed in the tank yesterday, $\frac{2}{5}(25)=10$ fish were placed in the tank yesterday. There are 40 fish now, so before the 10 were added there were 40 − 10 = 30. The number of fish increased by $\frac{10}{30}=\frac{1}{3}$ of the original number of fish.

If x is even and y is odd, what is $4x + yx + y^2$?

(A) Definitely odd

(B) Definitely even

(C) Could be either odd or even

Answer: (A) Definitely odd

$$\begin{aligned} 4x + xy + y^2 &= 4(\text{Even}) + (\text{Even})(\text{Odd}) + (\text{Odd})^2 \\ &= \text{Even} + \text{Even} + \text{Odd} \\ &= \text{Odd} \end{aligned}$$

If all variables are integers, and the product $abcd$ is odd, what is $abc + bcd + acd$?

(A) Definitely odd

(B) Definitely even

(C) Could be either odd or even

Answer: (A) Definitely odd

Since the product *abcd* is odd, none of the variables can be even. A single even integer would introduce a factor of 2 that would make the resulting product even. So, each variable is odd.

$$\begin{aligned} abc + bcd + acd &= (\text{Odd} \times \text{Odd} \times \text{Odd}) + (\text{Odd} \times \text{Odd} \times \text{Odd}) + (\text{Odd} \times \text{Odd} \times \text{Odd}) \\ &= \text{Odd} + \text{Odd} + \text{Odd} \\ &= (\text{Odd} + \text{Odd}) + \text{Odd} \\ &= \text{Even} + \text{Odd} \\ &= \text{Odd} \end{aligned}$$

If m, n, and p are consecutive integers such that $m < n < p$, what is $m^2 + n^2 + np$?

(A) Definitely odd

(B) Definitely even

(C) Could be either odd or even

Answer: (A) Definitely odd

Since consecutive integers alternate O, E, O, E...etc., there are two cases:

m	n	p	$m^2 + n^2 + np$
E	O	E	$E^2 + O^2 + OE = E + O + E = O$
O	E	O	$O^2 + E^2 + EO = O + E + E = O$

Both wind up odd.

x is a prime number and y is a positive multiple of x. What is $x - y$?

(A) Definitely odd

(B) Definitely even

(C) Could be either odd or even

Answer: (C) Could be either odd or even

If x is a prime number, it could be even (2 only) or odd (all the other prime possibilities). If y is a positive multiple of x, it could be either an even number or an odd number times x. Thus, there are several cases:

x	y	Comment	$x - y$
2	E	y is mult of $x = 2$, so y must be even.	$E - E = E$
O	O	y could be 3 times x, so odd.	$O - O = E$
O	E	y could be 4 times x, so even.	$O - E = O$

If j and k are integers such that j^2 is even and k^3 is odd, what is $2k + 5jk + 3j$?

(A) Definitely odd

(B) Definitely even

(C) Could be either odd or even

Answer: (B) Definitely even

Since j is an integer and j^2 is even, j itself must be even.

Since k is an integer and k^3 is odd, k itself must be odd.

$$\begin{aligned} 2k+5jk+3j &= 2(\text{Odd})+5(\text{Even})(\text{Odd})+3(\text{Even}) \\ &= \text{Even}+\text{Even}+\text{Even} \\ &= \text{Even} \end{aligned}$$

If f is an integer, what is $f^2 + 15f + 50$?

(A) Definitely odd

(B) Definitely even

(C) Could be either odd or even

Answer: (B) Definitely even

If f is even, then $f^2 + 15f + 50 = E^2 + 15E + 50 = E + E + E = E$

If f is odd, then $f^2 + 15f + 50 = \text{Odd}^2 + 15(\text{Odd}) + 50 = O + O + E = E$

Either outcome is even.

Alternatively, factor:

$$f^2 + 15f + 50 = (f + 5)(f + 10) = (f + \text{Odd})(f + \text{Even})$$

Since one term adds Odd and the other adds Even, there will always be one even and one odd term in the product. Both (E)(O) and (O)(E) are even.

If x and y are integers, $xy = z$ and $x + y = 11$, what is $z^2 - 2z$?

(A) Definitely odd

(B) Definitely even

(C) Could be either odd or even

Answer: (B) Definitely even

Since $x + y = 11 =$ odd, x and y must be odd and even, respectively, or vice versa. x and y cannot both be odd or both be even, as $x + y$ would be even in those cases.

The product xy will always be even, as both (E)(O) and (O)(E) are even. Thus, z is even.

$z^2 - 2z = E^2 - 2E = E - E = E$

Is zero odd, even, or neither? Why?

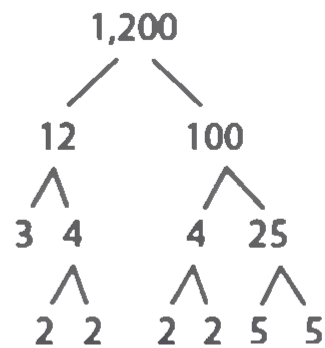
Answer: Zero is even.

Why?

(1) An even number is a number that, when divided by 2, yields an integer result. Note that $\frac{0}{2}=0=$ an integer.

(2) On the number line, odd and even integers alternate. That is, every even integer is between two odd integers. Every odd integer is between two even integers. Keeping this pattern, the even integer zero is between the odd integers –1 and +1. The odd integer 1 is between the even integers 0 and 2.

If x and y are positive integers and $3^x y^2 = 1{,}200$, what is xy?



Answer: 20

From the factor tree for 1,200 and the given constraint, $3^x y^2 = 3^1 2^4 5^2$.

y cannot have a factor of 3, as y^2 would then have two factors of 3, but there is only one factor of 3 in 1,200. Thus, $3^x = 3^1$, and $x = 1$. Consider the remaining terms and factors:

$y^2 = 2^4 5^2$

$y^2 = (2^2)^2 5^2$

$y^2 = (2^2 \times 5)^2$

$y = 2^2 \times 5 = 20$

So $x = 1$, $y = 20$, and $xy = 20$.

Round the following numbers to the <u>nearest 0.01</u>.

(A) 1.378

(B) 0.2547

(C) 9.355

Answer: (A) 1.38 (B) 0.25 (C) 9.36

When rounding to the nearest 0.01 (hundredth),

- Round UP when the thousandths digit is 5, 6, 7, 8, or 9.
- KEEP the hundredths digit as-is when the thousandths digit is 0, 1, 2, 3, or 4. (This is just like truncating every digit after the 0.01 place.)

(A) The thousandths digit is 8, so round the 7 in the hundredths place UP to 8. Result: 1.38

(B) The thousandths digit is 4, so truncate every digit after the hundredths place. Result: 0.25

(C) The thousandths digit is 5, so round the 5 in the hundredths place UP to 6. Result: 9.36

What is 0.472 as a fully reduced fraction?

Answer: $\frac{59}{125}$

Use the place value of the last digit in the decimal as the denominator: Here, 0.472 ends with the thousandths place, so put 1000 in the denominator. Put the decimal's digits in the numerator. Then simplify. 472 is divisible by 8, as is 1,000.

$$0.472 = \frac{472}{1,000} = \frac{472 + 8}{1,000 + 8} = \frac{59}{125}$$

Simplify: $\frac{12}{17}-\frac{1}{2}+\frac{27}{34}$

Answer: 1

$$
\begin{aligned}
\frac{12}{17}-\frac{1}{2}+\frac{27}{34} &= \frac{2\times 12}{2\times 17}-\frac{17\times 1}{17\times 2}+\frac{27}{34} \\
&= \frac{24}{34}-\frac{17}{34}+\frac{27}{34} \\
&= \frac{24-17+27}{34} \\
&= \frac{34}{34} \\
&= 1
\end{aligned}
$$

Simplify, and express the answer as a mixed fraction.

$$2\,{}^{7}\!/_{8} + 30\,{}^{1}\!/_{4}$$

Answer: $33\frac{1}{8}$

It is simplest to sum the whole number and the fractional parts of these terms separately:

$$
\begin{aligned}
2\tfrac{7}{8}+30\tfrac{1}{4} &= (2+30)+\left(\frac{7}{8}+\frac{1}{4}\right)\\
&= 32+\left(\frac{7}{8}+\frac{2}{8}\right)\\
&= 32+\frac{9}{8}\\
&= 32+1\tfrac{1}{8}\\
&= 33\tfrac{1}{8}
\end{aligned}
$$

Solve for x.

$$|3 - x| = 5$$

Answer: $x = -2$ or 8

$|3-x|=5$, so either $(3 - x) = 5$ or $(3 - x) = -5$. Solve both.

$$\begin{aligned} (3-x)&=5 \\ -x&=2 \\ x&=-2 \end{aligned} \qquad \text{or} \qquad \begin{aligned} (3-x)&=-5 \\ -x&=-8 \\ x&=8 \end{aligned}$$

Solve for x.

$$17-|2x+11|=-2$$

Answer: $x = 4$ or -15

First, isolate the absolute value sign.

$$17-|2x+11|=-2$$
$$19-|2x+11|=0$$
$$19=|2x+11|$$

So, either $(2x + 11) = 19$ or $(2x + 11) = -19$. Solve both.

$$\begin{aligned}(2x+11)&=19\\ 2x&=8\\ x&=4\end{aligned} \quad \text{or} \quad \begin{aligned}(2x+11)&=-19\\ 2x&=-30\\ x&=-15\end{aligned}$$

Each is a valid solution of the original equation, as we can see by plugging back in.

Solve for x.

$$\left|x^2+4\right| \leq 4$$

Answer: $x = 0$

$|something| \leq 4$ **means that** $-4 \leq something \leq 4$.

So, $|x^2+4| \leq 4$ means that $-4 \leq x^2 + 4 \leq 4$. Subtract 4 *from all three sides of the inequality* to isolate the x term: $-8 \leq x^2 \leq 0$. But the fact that x^2 is greater than a negative number is nothing new. A squared term can never be less than 0. Thus, the only valid solution to the inequality is $x = 0$.

Solve for x.

$$\left|\frac{1}{2}x - 12\right| = 14$$

Answer: $x = -4$ or 52

$\left|\frac{1}{2}x-12\right|=14$, so either $\left(\frac{1}{2}x-12\right)=14$ or $\left(\frac{1}{2}x-12\right)=-14$. Solve both.

$$\left(\frac{1}{2}x-12\right)=14 \qquad\qquad \left(\frac{1}{2}x-12\right)=-14$$

$$\frac{1}{2}x=26 \qquad \text{or} \qquad \frac{1}{2}x=-2$$

$$x=52 \qquad\qquad x=-4$$

Each is a valid solution of the original equation, as we can see by plugging back in.

If $n \otimes k = (n+k)^2$, what is $\frac{2}{9} \otimes \left(\frac{5 \otimes 3}{4 \otimes 2}\right)$?

Answer: 4

Deal with the expression in the parentheses first. Remember that fraction bars are like parentheses, too, so first solve $5\otimes 3$ and $4\otimes 2$ separately.

$$\begin{aligned}\frac{2}{9}\otimes\left(\frac{5\otimes 3}{4\otimes 2}\right)&=\frac{2}{9}\otimes\left(\frac{(5+3)^2}{(4+2)^2}\right)\\&=\frac{2}{9}\otimes\left(\frac{64}{36}\right)=\frac{2}{9}\otimes\left(\frac{16}{9}\right)\\&=\left(\frac{2}{9}+\frac{16}{9}\right)^2=\left(\frac{18}{9}\right)^2\\&=2^2=4\end{aligned}$$

If $S_n = n^2 + n - 1$, what is $S_6 - S_3$?

Answer: 30

$S_6 = 6^2 + 6 - 1 = 36 + 6 - 1 = 41$

$S_3 = 3^2 + 3 - 1 = 9 + 3 - 1 = 11$

$S_6 - S_3 = 41 - 11 = 30$

$A_n = 3A_{n-1} + 4$ for all $n \geq 2$. If $A_3 = 34$, what is A_1?

356

Answer: 2

The subscript on A denotes a term's order in the sequence. If A_n is "this term," then A_{n-1} is "the previous term." Each term in this sequence is based on the previous term, so given any term you can plug to get the next term, or backsolve to get the previous term.

$A_3 = 34 = 3A_2 + 4$, so $30 = 3A_2$, and $A_2 = 10$.

$A_2 = 10 = 3A_1 + 4$, so $6 = 3A_1$, and $A_1 = 2$.

$$S_n = \frac{1}{2}S_{n-1} + S_{n-2} + 4 \text{ for all } n \geq 3.$$

If $S_1 = 1$ and $S_2 = 2$, what is S_4?

Answer: 9

The subscript on S denotes a term's order in the sequence. If S_n is "this term," then S_{n-1} is "the previous term" and S_{n-2} is "the term before that." Plug in to solve for each term in succession.

$$S_3 = \frac{1}{2}S_2 + S_1 + 4 = \frac{1}{2}(2) + (1) + 4 = 6$$

$$S_4 = \frac{1}{2}S_3 + S_2 + 4 = \frac{1}{2}(6) + (2) + 4 = 9$$

Each number in a sequence is 9 more than the previous term. If the 11th number in the sequence is 49, what is the 3rd number in the sequence?

Answer: −23

There are 8 "jumps" from the 3rd term to the 11th term, so think of the 3rd term as
8 jumps of 9 "back" or "down" from the 11th term.

3rd term = 11th term − (8 jumps) + (9 per jump) = 49 − 72 = −23

In a sequence, $A_n = 4^n - 1$

Quantity A	Quantity B
The units digit of A_{17}	The units digit of A_{93}

Answer: (C) The two quantities are equal.

The units digits of the powers of 4 exhibit a repeating pattern. The powers are 4, 16, 64, 256, 1,024, etc., so the units digits pattern is {4, 6, 4, 6, 4, etc.} Because 1 is subtracted from each term, the units digits for A_n will have the pattern {3, 5, 3, 5, 3, etc.} When the subscript is odd, the units digit of A_n is 3. When the subscript is even, the units digit of A_n is 5.

In both quantities, the subscript is odd, so the units digits are equal.

If $f(x) = \dfrac{3x+2}{5x}$, what is f(20)?

360

Answer: 0.62 or $\frac{31}{50}$

$$f(20)=\frac{3(20)+2}{5(20)}=\frac{60+2}{100}=\frac{62}{100}=0.62$$

If $g(x) = 4x$ and $f(x) = x^3 + 5$, what is $g(f(5))$?

361

Answer: 520

Start from the inside and work outward: First input 5 to the f function, then input *the result* to the g function.

$f(5) = (5)^3 + 5 = 125 + 5 = 130$
$g(f(5)) = g(130) = 4(130) = 520$

Solve for k.

$$3|4-k|=45$$

Answer: $k = -11$ or 19

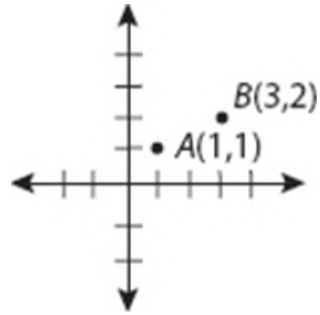
First, isolate the absolute value sign.

$$3|4-k| = 45$$
$$|4-k| = 15$$

So, either $(4 - k) = 15$ or $(4 - k) = -15$. Solve both.

$$\begin{aligned} (4-k) &= 15 \\ -k &= 11 \\ k &= -11 \end{aligned} \quad \text{or} \quad \begin{aligned} (4-k) &= -15 \\ -k &= -19 \\ k &= 19 \end{aligned}$$

Each is a valid solution of the original equation, as we can see by plugging back in.



If point C has integer coordinates such that $0 \leq x \leq 5$ and $0 \leq y \leq 5$, how many different triangles ABC can be constructed?

Answer: 33

Point C can be anywhere in the grid of integer coordinate points given by the inequalities. There are 6 possible x coordinates and 6 possible y coordinates (0, 1, 2, 3, 4, or 5). Thus, there are (6)(6) = 36 possible integer coordinate points in the region. However, to create a triangle ABC, Point C cannot coincide with either Point A or Point B, nor lie on the line joining them. Points (1,1) and (3,2) are invalid.

Since the slope of the line joining the points is $\frac{2-1}{3-1}=\frac{1}{2}$, the only other integer coordinate point on the line (and on the grid) is (5,3), as we can see by drawing the line. So the possible locations for Point C are 36 – 3 = 33 in number.

An airline has sold 20% more tickets for a flight than there are seats available. If 10% of the ticket holders do not arrive at the airport for the flight, what fraction of the ticket holders who do arrive at the airport will definitely not get a seat on the flight?

Answer: 2/27

Because no actual numbers are given, just percents, a Smart Number of 100 is useful. Suppose there are 100 seats on the flight, and the airline has sold 20% more than 100, or 120 tickets. Of those 120 ticket holders, 10% or 12 ticket holders do not show up for the flight. Thus, 120 – 12 = 108 people arrive at the airport, but 108 – 100 = 8 of them will definitely not get a seat on the flight.

$$\frac{\text{will not get seat}}{\text{arrive at airport}} = \frac{8}{108} = \frac{2}{27}$$

(It is possible that even *more* ticket holders will not get a seat—maybe some of them don't make it through security—but we know that 8 ticket holders will definitely not get seats.)

Quantity A	Quantity B
The area of a rectangle with length 3 and width 5.	The area of a parallelogram that has a pair of parallel sides 5 in length with a distance of 3 between them.

Answer: (C) The two quantities are equal.

Quantity A: Area of a rectangle = $wL = (5)(3) = 15$.

Quantity B: Area of a parallelogram = bh, where b is the length of a pair of parallel "base" sides and h is the perpendicular distance between these bases. Here, the area of the parallelogram = $bh = (5)(3) = 15$.

Note: Figure not drawn to scale

<u>Quantity A</u> | <u>Quantity B</u>

The area of rectangle *A* above.

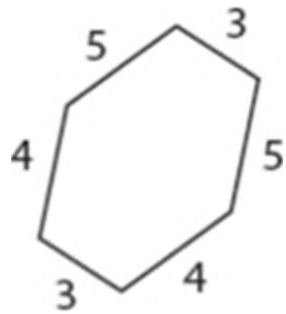
The area of trapezoid *B* above.

Answer: (B) Quantity B is greater.

Quantity A: Area of rectangle $A = wL = (4)(14) = 56$

Quantity B: Area of trapezoid $B = \frac{b_1 + b_2}{2} \times h = \frac{8+12}{2} \times 7 = \frac{20}{2} \times 7 = 70$

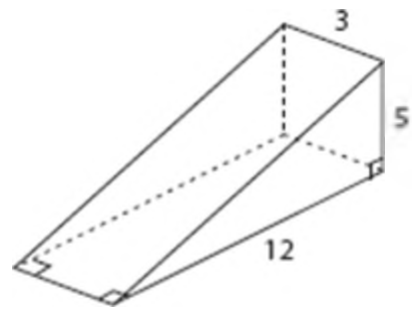
What is the perimeter of the polygon below?



Answer: 24

The perimeter of a polygon is the sum of all side lengths. Starting at the top and going clockwise, Perimeter $= 3 + 5 + 4 + 3 + 4 + 5 = 24$.

What is the surface area of the wedge-shaped solid below? (The base and back rectangles are at a right angle to one another.)



Answer: 150

First, note that the front and back are 5–12–3 right triangles, so the sloped top edges of the wedge are 13 long.

Bottom Rectangle: Area = wL = (3)(12) = 36

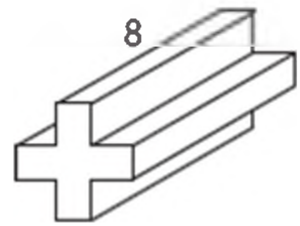
Front Triangle: $\text{Area} = \frac{1}{2}bh = \frac{1}{2}(12)(5) = 30$

Back Triangle: Area = $\text{Area} = \frac{1}{2}bh = \frac{1}{2}(12)(5) = 30$

Vertical Rectangle: Area = wL = (5)(3) = 15

Sloped Top Rectangle: Area = wL = (13)(3) = 39

Total Surface Area = 36 + 30 + 30 + 15 + 39 = 150



The figure above shows a symmetrical solid, similar to an extruded "+" sign. All surfaces of the solid meet at right angles, and each edge on the front "+" face is 1 long. What is the surface area of the solid?

Answer: 106

The surface area consists of two "+" shaped faces (front and back), and twelve 1 by 8 rectangles (sides).

The front and back faces can be seen as five 1 by 1 squares:

Front and Back: $(5)(1)(1)$ each, so $5 + 5 = 10$ total.
Sides: (12 side rectangles)$(1)(8) = 96$

The total surface area is $10 + 96 = 106$.

Quantity A	Quantity B
The sum of the interior angles of a rhombus.	The sum of the interior angles of a square.

Answer: (C) The two quantities are equal.

The side lengths are irrelevant, as are the different individual angles. The sum of the interior angles of any polygon only depends on number of sides.

Sum of the interior angles = $180°(n - 2)$, where n = the number of sides in the polygon.

Both a rhombus and a square have 4 sides, so they have the same interior angle sum: $180°(4 - 2) = 360°$.

The rhombus above has perimeter of 40.

Quantity A	Quantity B
The area of the rhombus	102

Answer: (B) Quantity B is greater.

A rhombus has two sets of parallel sides, and four sides of equal length. Each side of this rhombus is $40/4 = 10$.

The area of a rhombus in general is bh, where $b = 10$ and h is < 10 if the rhombus "leans" at all, i.e. bottom left and top right angles less than 90°. In these cases, bh = (10)(less than 10) = less than 100.

The area of the rhombus is maximized if all the angles are equal to 90° (i.e. if the shape is a square, which is a subset of rhombus). In that case, the area = s^2 = $(10)(10) = 100$.

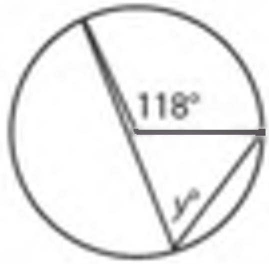
The area of the rhombus cannot equal or exceed 102.

Quantity A	Quantity B
The radius of a circle with area 8π	The circumference of a circle with radius $\frac{3}{2\pi}$.

Answer: (B) Quantity B is greater.

Quantity A: Area = $\pi r^2 = 8\pi$, so $r^2 = 8$. 8 is less than 9, so the radius is less than $\sqrt{9}$ which is less than 3.

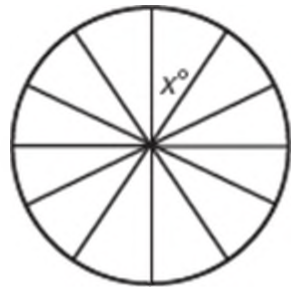
Quantity B: Circumference $= 2\pi r = 2\pi\left(\frac{3}{2\pi}\right) = 3$.



If the 118° angle is a central angle of the circle above, what is y?

Answer: 59

If the 118° angle is a central angle of the circle above, the $y°$ angle is an inscribed angle bounded by the same arc on the circle. For a given arc, an inscribed angle is always $\frac{1}{2}$ the central angle. $\frac{1}{2}(118)=59$.



A circular pizza is cut into 12 identical slices, as shown. What is x?

Answer: 30

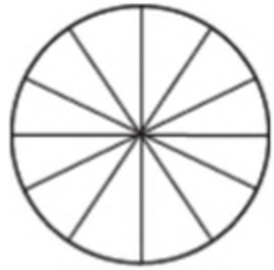
The point of each slice is a central angle of the circle. The ratio of the central angle to 360° is equal to the proportion of the circle enclosed.

Each slice = $\frac{1}{12}$ of the circle = $\frac{x}{360}$. Thus, $x=\frac{360}{12}=30$.

x is $\frac{1}{12}$ of the circle, so:

$$\frac{1}{12}=\frac{x}{360}$$

$$x=\frac{360}{12}=30$$



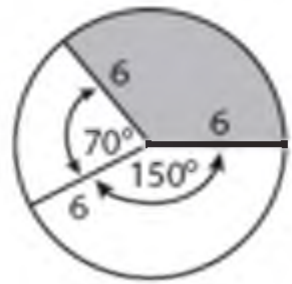
A circular pizza is cut into 12 identical slices, as shown. If each slice is 3π square inches in area, what is the diameter of the whole pizza?

Answer: 12 inches

Each of the 12 pieces is 3π square inches in area, so the area of the whole pizza is $(12)(3\pi) = 36\pi$ square inches.

Area of the circle $= \pi r^2 = 36\pi$. Thus $r = 6$ inches.

The diameter is twice the radius, or $(2)(6) = 12$ inches.



What is the area of the shaded sector of the circle above?

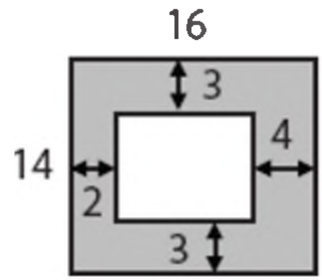
Answer: 14π

Because all three lines are the same length, meet at the same point, and extend to the circle edge, all must be radii of the circle (and the point at which they meet must be the center of the circle). If the radius is 6, the area of the whole circle is $\pi r^2 = 36\pi$.

The shaded sector has a central angle of $360° - 70° - 150° = 140°$.

$$\frac{\text{Sector Area}}{\text{Circle Area}} = \frac{\text{Sector Area}}{36\pi} = \frac{140°}{360°}$$

Thus, Sector $\text{Area} = \frac{140°}{360°} \times 36\pi = 14\pi$. .



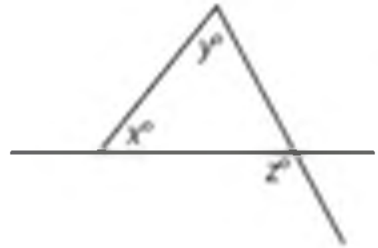
In the figure above, both shapes are rectangles, and all lines are either horizontal or vertical. What is the area of the shaded region?

Answer: 144

The area of the larger rectangle is $wL = (16)(14) = 224$

The area of the smaller rectangle is $wL = (16 - 2 - 4)(14 - 3 - 3) = (10)(8) = 80$.

The area of the shaded region is the difference between the larger and smaller rectangle areas: $224 - 80 = 144$.



Quantity A	**Quantity B**
$x + y$	z

Answer: (C) The two quantities are equal.

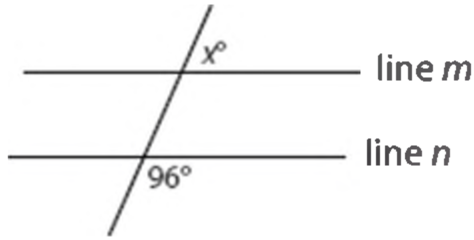
Label the bottom right angle of the triangle $b°$. From the sum of the angles in the triangle, it is known that $x + y + b = 180$, or $b = 180 - x - y$. From the sum of the angles that form a line, it is known that $b + z = 180$, or $b = 180 - z$. Setting these two expressions for b equal to one another:

$180 - x - y = 180 - z$

$-x - y = -z$

$x + y = z$

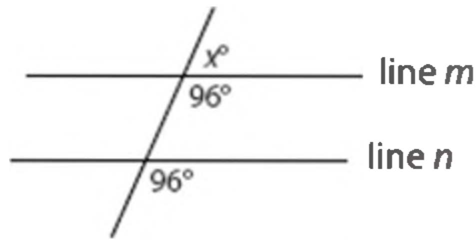
Or, note that z is an exterior angle of the triangle, which is always equal to the sum of the two opposite interior angles of the triangle, $x + y$.



In the figure above, lines *m* and *n* are parallel. What is *x*?

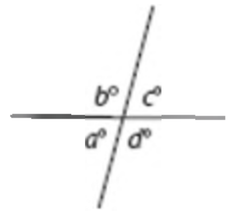
Answer: 84

Because lines m and n are parallel, the transversal crosses both at the same angle. Thus, the angle directly below $x°$ is also 96°.



Angle x and the newly labeled angle form a straight line, so their sum is 180: $x + 96 = 180$, so $x = 180 - 96 = 84$.

$b > c$



Quantity A	Quantity B
$\dfrac{a+b+c}{2}$	d

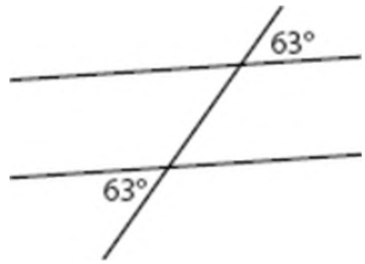
Answer: (D) The relationship cannot be determined from the information given.

Angles *a*, *b*, *c*, and *d* form a complete circle, or a total of 360°. Since *b* and *c* form a line, $b + c = 180$, so the $b > c$ constraint indicates that $b > 90$ while $c < 90$. Similarly, $d > 90$ while $a < 90$.

Quantity A: $\frac{a+b+c}{2} = \frac{360-d}{2} = 180 - \frac{d}{2}$

The two quantities would be equal if $180 - \frac{d}{2} = d$, or $180 = \frac{3d}{2}$, or $d = \frac{2}{3} \times 180 = 120$.

The two quantities are equal if $d = 120$.
If $90 < d < 120$, Quantity A is greater.
If $120 < d < 180$, Quantity B is greater.

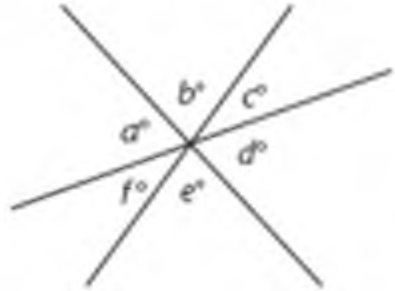


What other information can be inferred from the figure above?

Answer: The two almost-horizontal lines are parallel; all of the other angle measures can be determined.

The vertical angles (opposite the labeled $63°$ angles where two lines intersect), are also $63°$. The other angles form a line with one of these $63°$ angles (they are supplementary), and can be labeled $180 - 63 = 117°$.

Finally, because the transversal crosses both of the two almost-horizontal lines at the same angle, these two lines are parallel.



What is $a + c + e$?

Answer: 180

Notice that $b° = e°$, because these two angles are vertical angles, i.e. opposite each other where two lines intersect.

Thus, $a + c + e = a + c + b$. This is useful because the angles a, b, and c form a line, or are supplementary. The sum of such angles is 180.

If the equation of a line is $y = -3x + 6$, what is the x-intercept of the line?

Answer: x-intercept = 2

Watch out! This question is not about the y-intercept of the line, which is the constant +6 from the given line equation, given in standard slope-intercept form.

This question is about the x-intercept, which is where the line crosses the x-axis. At that point, the y coordinate is 0, so this question is really "What is x when $y = 0$?"

$y = -3x + 6$

$0 = -3x + 6$

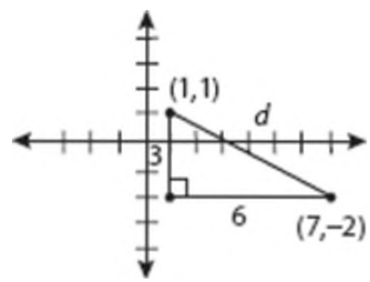
$-6 = -3x$

$x = 2$

What is the distance between $(1, 1)$ and $(7, -2)$ on the coordinate plane?

Answer: $3\sqrt{5}$

Draw the points on the coordinate plane, along with the right triangle for which the point to point distance is the hypotenuse.



The horizontal leg of the triangle is $|7 - 1| = 6$.

The vertical leg of the triangle is $|-2 - 1| = 3$.

By Pythagorean Theorem, the distance (hypotenuse) is $\sqrt{6^2+3^2} = \sqrt{36+9} = \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$.

Fiona bicycled 20 miles on a trail at 10 miles per hour, then walked on the same trail back to her starting place at 4 miles per hour. What was her average speed for the round trip, in miles per hour?

Answer: $\frac{40}{7}$ or $5\frac{5}{7}$

Average speed is total distance over total time.

Time spent bicycling: $\frac{20 \text{ miles}}{10 \text{ miles per hour}} = 2 \text{ hours}$

Time spent walking: $\frac{20 \text{ miles}}{4 \text{ miles per hour}} = 5 \text{ hours}$

Her total trip was 20 + 20 = 40 miles, and it took 2 + 5 = 7 hours.

Average rate for the round trip:

$\frac{40 \text{ miles}}{7 \text{ hours}} = \frac{40}{7}$ miles per hour $= 5\frac{5}{7}$ miles per hour

In a fruit salad, there are 7 grapes for every 2 oranges, and 3 oranges for every pineapple. If 4 pineapples are in the salad, how many grapes are in the salad?

Answer: 42

You can solve in steps:

There are 3 oranges for every pineapple, and 4 pineapples in the salad. There are (3)(4) = 12 oranges in the salad.

There are 7 grapes for every 2 oranges, so there are 7/2 grapes per orange. There are (7/2)(12) = 42 grapes in the salad.

Or, you can set up ratios and create a common term:

$$\begin{array}{ll}
\underline{\mathrm{G} : \mathrm{Or} : P} & \\
7 : 2 & \text{(multiply by 3)} \\
\underline{\quad\ 3 : 1} & \underline{\text{(multiply by 2)}} \\
21 : 6 : 2 & \text{(common term ratio)}
\end{array}$$

Since there are 4 pineapples, double this ratio to 42 : 12 : 4, and see that there are 42 grapes.

A soup recipe requires 2 cups of chopped tomatoes, 1 cup of white beans, 4 cups of chopped spinach, and 4 cups of broth. If there are 6 cups of white beans in a large batch of soup, how many cups of ingredients total are in the batch?

Answer: 66

The ratio of (tomatoes : beans : spinach: broth) is given as $(2x : 1x : 4x : 4x)$. The total number of cups of ingredients is $2x + 1x + 4x + 4x = 11x$.

In the large batch of soup, there are 6 cups of beans, so $x = 6$. Thus, there are $11x = 11(6)$ total cups of ingredients in the batch.

At a photography exhibit, the ratio of color photographs to black and white photographs displayed is 1 to 8. If there are 72 photographs of these two types in the exhibit, how many color photographs are on display?

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Answer: 8 (not 9!)

The ratio of (color : b&w) is given as (1 : 8), or using the unknown multiplier, ($1x$: $8x$). There are $1x + 8x = 9x = 72$ photographs on display, so $x = 8$.

Thus, there are $1x = 1(8)$ color photographs on display.

Be careful! The 1 : 8 ratio does not mean that 1/8 of the photographs are in color. It means that 1/9 of the total are color photographs and 8/9 of the total are black and white.

In a certain school building, $\frac{3}{7}$ of the classrooms are carpeted, and the remainder have tile floors. If 12 classrooms have tile floors, how many classrooms are in the school building?

Answer: 21

If $\frac{3}{7}$ of the classrooms are carpeted, then $1-\frac{3}{7}=\frac{4}{7}$ of the classrooms have tile floors. If x is the number of classrooms in the building:

$$\frac{4}{7}x=12\text{, so } x=12\times\frac{7}{4}=3\times7=21.$$

The ratio of boys to girls in a science class is 7 to 5. If there are 21 boys in the class, how many girls are in the class?

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Answer: 15

Set up a labeled proportion: $\frac{7 \text{ boys}}{5 \text{ girls}} = \frac{21 \text{ boys}}{x \text{ girls}}$

Cross multiply: $7x = 105$

$x = 15$

The ratio of adults to children at a picnic is 4 to 3. If there are 49 people at the picnic, how many of the people are adults?

Answer: 28

Use the unknown multiplier, where x is an integer: $\frac{\text{Adults}}{\text{Children}} = \frac{4x}{3x}$

$49 = \text{Adults} + \text{Children} = \text{Total}$

$49 = 4x + 3x$

$49 = 7x$

$x = 7$

There are $4x = 4(7) = 28$ adults at the picnic.

Harvey checked out books from the library, and the ratio of hardcover books to paperback books was 5 to 4. If the library permits patrons to check out no more than 30 books at a time, what is the maximum possible number of paperback books Harvey checked out?

Answer: 12

Use the unknown multiplier, where x is an integer: $\frac{\text{Hardcover}}{\text{Paperback}} = \frac{5x}{4x}$

$30 \geq \text{Hardcover} + \text{Paperback} = \text{Total}$

$30 \geq 5x + 4x$

$30 \geq 9x$

Because x must be an integer, the greatest possible value for x is 3. Harvey could have checked out no more than $4x = 4(3) = 12$ paperback books.

The members of a running club ran a total of 1,226 miles. If no member of the club ran less than 26 miles, what is the maximum number of running club members?

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Answer: 47

To maximize the number of runners, we need to minimize the number of miles each member ran.

Miles per runner	×	Runners	=	Total miles
At LEAST 26 minimize	×	At MOST ??? maximize	=	EXACTLY 1,226 constant

Number of Runners = $\frac{1{,}226}{\text{at least } 26}$ = at most 47.154. Since there obviously cannot be a fractional number of running club members, the maximum is 47.

A presentation will be given on Thursday then repeated on Friday and Saturday, in a room that can accommodate 40 audience members. Of the 110 people to be scheduled to attend the presentation, 12 prefer to attend on Thursday, 18 prefer to attend on Friday, and 80 prefer to attend the presentation on Saturday. What is the minimum number of people who will <u>not</u> get to attend the presentation on their preferred day?

Answer: 40

Only 12 and 18 people, respectively, prefer to attend the presentation on Thursday and Friday. The room can accommodate these people, so they will get to attend on their preferred day.

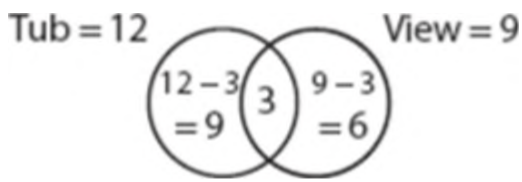
The limiting factor is Saturday: 80 people – 40 seats = 40 people who will have to attend on either Thursday or Friday instead of their preferred day.

At a resort, 1/2 of the rooms have a lake view and 2/3 of the rooms have a hot tub. If 1/3 of the rooms with a lake view have a hot tub, then rooms that have neither a lake view nor a hot tub are what fraction of the total number of rooms at the resort?

Answer: 0

Use a Smart Number and a Venn Diagram. 18 is a Smart Number to use for the total number of rooms, as it is a multiple of all the fraction denominators.

"1/2 of the rooms have a lake view": (1/2)(18) = 9
"2/3 of the rooms have a hot tub": (2/3)(18) = 12
"1/3 of the rooms with a lake view have a hot tub": (1/3)(9) = 3



Place the 3 in the middle first, then work out the outer numbers.
There are 9 + 3 + 6 = 18 rooms that have a hot tub, a view, or both. Thus, there are 18 − 18 = 0 rooms that have neither.

If it does not rain today, there is a 75% chance that the high temperature will be greater than 90°F. If it does rain today, there is a 50% chance that the high temperature will be greater than 90°F. If there is a 40% chance of rain, what is the probability that the high temperature will be greater than 90°F?

Answer: 65%

Chance of rain and high temperature above 90°F: $(0.40)(0.50) = 0.2$

Chance of NO rain and high temperature above 90°F: $(1 - 0.40)(0.75) = (0.6)(0.75) = 0.45$

Chance of high temperature above 90°F: $0.2 + 0.45 = 0.65 = 65\%$

The probability of flipping at least one head in a series of flips of a fair coin is calculated to be $\frac{15}{16}$. How many times will the coin be flipped?

Answer: 4

"Flipping at least one head" will NOT happen if all the coin flips produce tails. So if the probability of at least one head is 15/16, the probability of *all* tails is 1 – 15/16 = 1/16. The probability of flipping tails...

...once is

...twice in a row is

...three times in a row is

...four times in a row is

The coin would need to be flipped 4 times for the probability of flipping at least one head to be 15/16.

534,360 people travel through a train station per year. What is the average number of travelers through the train station per hour? (Assume the station operates continuously 365 days a year; be ready to use a calculator.)

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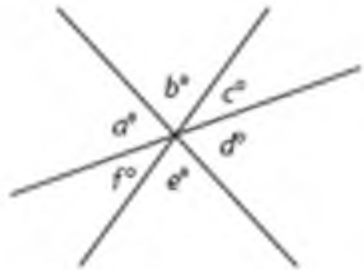
Answer: 61

Use the calculator.

$$\frac{534{,}360\text{ people/year}}{365\text{ days/year}} = 1{,}464\text{ people/day}$$

$$\frac{1{,}464\text{ people/day}}{24\text{ hours/day}} = 61\text{ people/hour}$$

This is a tough one for mental math, but in a pinch you could estimate.



Quantity A	Quantity B
$f + b + d$	$a + e$

Answer: (A) Quantity A is greater.

Notice that $b° = e°$, because these two angles are vertical angles, i.e. opposite each other where two lines intersect.

Thus, $f + b + d = f + e + d$. This is useful because the angles f, e, and d form a line, or are supplementary. The sum of such angles is 180.

The same could be said for angles a, f, and e. Since $a + f + e = 180$ and the figure indicates that $f > 0$, then $a + e < 180$.

Quantity A: $f + b + d = 180$
Quantity B: $a + e < 180$

Quantity A	Quantity B
$\sqrt{2} \times \sqrt{2}$	$\sqrt{2+2}$

Answer: (C) The two quantities are equal.

Quantity A: $\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2} = \sqrt{4} = 2$

Quantity B: $\sqrt{2+2} = \sqrt{4} = 2$

Quantity A	Quantity B
$\sqrt{6}+\sqrt{3}$	$\sqrt{6+3}$

Answer: (A) Quantity A is greater.

Quantity A: $\sqrt{6}+\sqrt{3} \approx 2.45+1.7=4.15$

Quantity B: $\sqrt{6+3}=\sqrt{9}=3$

In Quantity A, take the square root first, then add. In Quantity B, add first, then take the square root. The different order of operations leads to different values.

What is the interquartile range for the set {2, 3, 5, 7, 11, 13, 17, 19, 23}?

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Answer: 14

The interquartile range is the difference between the 1st quartile and the 3rd quartile of the set, or $Q_3 - Q_1$. There are 9 terms in the set, so the 5th (highest or lowest) is the median, which is also called the 2nd quartile. Median = $Q_2 = 11$.

Q_3 is the median of the upper half of the data, *not including the median of the overall set*. These upper terms are {13, 17, 19, 23}, so their median is the average of the two middle terms 17 and 19. $Q_3 = 18$.

Q_1 is the median of the lower half of the data, *not including the median of the overall set*. These lower terms are {2, 3, 5, 7}, so their median is the average of the two middle terms 3 and 5. $Q_1 = 4$.

The interquartile range is $Q_3 - Q_1 = 18 - 4 = 14$.

Simplify: $\left[(51+3+4)+7\right]+(4-7)$

Answer: 0

Remember PEMDAS order of operations: **P**arentheses, **E**xponents, **M**ultiplication/**D**ivision, **A**ddition/**S**ubtraction

Solve the innermost parentheses first. Within $(51 \div 3 + 4)$, divide before adding: $51 \div 3 = 17$ then add 4 to get 21.

Now, the expression is $[21 \div 7] + (4 - 7)$. Solve within the other parentheses: $[3] + (-3)$. Finally, add: $[3] + (-3) = 0$.

Which of the following are equal to one another? Select all that apply.

[A] $3 - 8 - 4 - 9$

[B] $3 - 8 - (4 - 9)$

[C] $3 - (8 - 4) - 9$

[D] $3 - (8 - 4 - 9)$

[E] $(3 - 8) - (4 - 9)$

Answer: [B] and [E]

[A] $3 - 8 - 4 - 9 = -18$

[B] $3 - 8 - (4 - 9) = -5 - (-5) = -5 + 5 = 0$

[C] $3 - (8 - 4) - 9 = 3 - (4) - 9 = -10$

[D] $3 - (8 - 4 - 9) = 3 - (-5) = 3 + 5 = 8$

[E] $(3 - 8) - (4 - 9) = (-5) - (-5) = -5 + 5 = 0$

In the PEMDAS order of operations (**P**arentheses, **E**xponents, **M**ultiplication/**D**ivision, **A**ddition/**S**ubtraction), addition and subtraction are equally ranked. Parentheses take precedence, which (for example) doesn't make any difference between $3 - 8$ and $(3 - 8)$, but does change things between $-4 - 9$ and $-(4 - 9)$, because the subtraction of -9 is different from the subtraction of just 9.

Solve for x and y:

$$3x + 2y = -4$$

$$5x - 2y = 20$$

Answer: $x = 2$ and $y = -5$

Because the coefficients of the y term are the same number (2) but with opposite signs, add the two equations together, eliminating the y terms.

$$\begin{array}{l} 3x+2y=-4 \\ \underline{5x-2y=20} \\ 8x+0y=16 \end{array}$$

Then, plug $x = 2$ into either original equation.

$$\begin{aligned} x&=2 \\ 5(2)-2y&=20 \\ -2y&=20-10=10 \\ y&=-5 \end{aligned}$$

Simplify:

$$\frac{8x+1}{3}-\frac{x-4}{2}$$

Answer: $\frac{13x+14}{6}$

Remember that fraction bars work like parentheses. Write the expression on your paper with parentheses around the whole numerators:

$$\begin{aligned}\frac{(8x+1)}{3}-\frac{(x-4)}{2} &= \frac{2}{2}\times\frac{(8x+1)}{3}-\frac{3}{3}\times\frac{(x-4)}{2}\\ &= \frac{2(8x+1)}{6}-\frac{3(x-4)}{6}\\ &= \frac{2(8x+1)-3(x-4)}{6}=\frac{(16x+2)-(3x-12)}{6}\\ &= \frac{16x+2-3x+12}{6}=\frac{13x+14}{6}\end{aligned}$$

If $x \neq \frac{1}{5}$, solve for x.

$$\frac{5x - 2(7 - x)}{5x - 1} = 2$$

Answer: $x = -4$

Remember that fraction bars work like parentheses. Write the fraction on your paper with parentheses around the whole denominator. Doing so makes it easier to follow what happens when you multiply both sides of the equation by $(5x - 1)$:

$$\frac{5x-2(7-x)}{(5x-1)}=2 \quad \longrightarrow \quad 5x-2(7-x)=2(5x-1)$$

Distribute and solve:

$$\begin{aligned} 5x-(14-2x)&=10x-2 \\ 5x-14+2x&=10x-2 \\ 7x-14&=10x-2 \\ -12&=3x \\ x&=-4 \end{aligned}$$

Simplify:

$$\frac{x-2}{2}+\frac{2-x}{4}$$

Answer:

Remember that fraction bars work like parentheses. Write the expression on your paper with parentheses around the whole numerators:

$$\begin{aligned}\frac{(x-2)}{2}+\frac{(2-x)}{4}&=\frac{2}{2}\times\frac{(x-2)}{2}+\frac{(2-x)}{4}\\&=\frac{2(x-2)}{4}+\frac{(2-x)}{4}=\frac{2(x-2)+(2-x)}{4}\\&=\frac{2x-4+2-x}{4}=\frac{x-2}{4}\end{aligned}$$

Distribute (FOIL) the expression:

$$(x - 5)\ (y + 7)$$

Answer: $xy + 7x - 5y - 35$

First: $(x)(y) = xy$

Outer: $(x)(7) = 7x$

Inner: $(-5)(y) = -5y$

Last: $(-5)(7) = -35$

Which of the following cannot be factored? Select <u>all</u> that apply.

[A] $x^2 + 2x - 8$

[B] $x^2 - 2x - 8$

[C] $x^2 + 2x + 8$

[D] $x^2 + 6x - 8$

[E] $x^2 - 6x - 8$

[F] $x^2 + 6x + 8$

Answer: [C], [D], and [E]

[A] $x^2 + 2x - 8 = (x + 4)(x - 2)$

[B] $x^2 - 2x - 8 = (x - 4)(x + 2)$

[C] CANNOT FACTOR $x^2 + 2x + 8$. The +8 demands same sign factors of 8, while the $+2x$ implies these factors must be positive and sum to 2. There is not an integer factor pair of 8 that sums to 2. (In fact, there's not even a non-integer factor pair that works.)

[D] CANNOT FACTOR $x^2 + 6x - 8$. The –8 demands opposite sign factors of 8, while the $+6x$ implies these factors must differ in absolute value by 6. There is not an integer factor pair of 8 that differs by 6.

[E] CANNOT FACTOR $x^2 - 6x - 8$. The –8 demands opposite sign factors of 8, while the $-6x$ implies these factors must differ in absolute value by 6. There is not an integer factor pair of 8 that differs by 6.

[F] $x^2 + 6x + 8 = (x + 4)(x + 2)$

If $x \neq 2$, solve for x:

$$\frac{x^2 - 3 - 10}{x + 2} = 0$$

Answer: $x = 5$

$$\frac{x^2-3-10}{x+2}=\frac{(x-5)(x+2)}{x+2}=x-5=0$$

Thus, $x = 5$. Note that $x = -2$ would be a solution to the quadratic in the numerator. However, the denominator of $x + 2$ would be zero in this case, and the fraction would be undefined. There is only one valid solution.

Solve for z:

$$z^3 - 2z^2 + z = 0$$

Answer: $z = 0$ or 1

Factor:

$$
\begin{aligned}
z^3 - 2z^2 + z &= 0 \\
z\left(z^2 - 2z^1 + 1\right) &= 0 \\
z(z-1)(z-1) &= 0 \\
z(z-1)^2 &= 0
\end{aligned}
$$

There are two solutions: $z = 0$ or $z = 1$

Factor:

$t^5 + 4t^3$

Answer: $t^3(t^2 + 4)$

The largest factor common to both terms is t^3, so factor it out.

$t^5 + 4t^3 = t^3(t^2 + 4)$

The expression remaining in the parentheses cannot be factored further.

Solve for x:

$$x(x + 8) = 4(2x + 1)$$

Answer: $x = 2$ or -2

Distribute and simplify:

$$x(x+8)=4(2x+1)$$
$$x^2+8x=8x+4$$
$$x^2=4$$

From here, you could either remember that even exponents indicate two solutions (here, positive and negative 2) or you could factor:

$$x^2=4$$
$$x^2-4=0$$
$$(x+2)(x-2)=0$$

$x < 8$

<u>Quantity A</u>	<u>Quantity B</u>
x	$\lvert x \rvert$

Answer: (D) The relationship cannot be determined from the information given.

If $x < 8$, x could be positive (e.g. $x = 5$), negative (e.g. $x = -10$), or zero.

If x is positive, the two quantities are equal, because absolute value signs don't do anything to a positive number. Using $x = 5$ as an example, $|5| = 5$, so $|x| = x$. The same is true if $x = 0$.

If x is negative, Quantity A is negative and Quantity B is positive, so Quantity A < Quantity B. Using $x = -10$ as an example, $|-10| = 10$, and $10 > -10$, so $|x| > x$.

Solve for g:

$$|15 - 2g| = 31$$

Answer: $g = -8$ or 23

Solve for two cases separately. The expression inside the absolute value sign equals either +31 or −31.

$$\begin{aligned} 15-2g &= 31 \\ -2g &= 16 \\ g &= -8 \end{aligned} \qquad \text{OR} \qquad \begin{aligned} 15-2g &= -31 \\ -2g &= -46 \\ g &= 23 \end{aligned}$$

Each is a valid solution of the original equation, as we can see by plugging back in.

Solve for x:

$$-19 < 2x - 5 < 19$$

Answer: $-7 < x < 12$

For a compound inequality, any operation you do must be performed on every term of the inequality, not just the outside terms.

$-19 < 2x - 5 < 19$

$-14 < 2x < 24$

$-7 < x < 12$

Solve for x:

$$|2x+3|<17$$

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Answer: $-10 < x < 7$

The expression inside the absolute value sign is either non-negative (but less than 17), or negative (between -17 and 0). Combine these possibilities into one inequality and solve. For such a compound inequality, any operation you do must be performed on every term of the inequality, not just the outside terms.

$-17 < 2x + 3 < 17$

$-20 < 2x < 14$

$-10 < x < 7$

$-2 \leq x \leq 9$ and $-5 \leq y \leq 1$

Quantity A	**Quantity B**
xy	10

Answer: (D) The relationship cannot be determined from the information given.

Don't assume that the maximum xy value is $(9)(1) = 9$, just because those are the upper limits for x and y respectively. Both x and y have negative possible values, and (negative)(negative) = positive. In fact, $(-2)(-5) = 10$ is the maximum xy value. So Quantity A could be equal to or less than 10, and the relationship is not certain.

If $0 \le x \le 4$ and $-6 \le y \le -1$, what is the maximum value of xy?

Answer: 0

x can be zero or positive. y can only be negative. Since x and y cannot have the same sign, the product xy cannot be positive.

(positive x) (negative y) would yield a negative xy.

The maximum xy is $(x = 0)$ (negative y) $= 0$.

Which of the following are in the range of solutions for $x^2 - 25 < 0$? Select <u>all</u> that apply.

[A] –10

[B] –5

[C] –5/2

[D] 0

[E] 4

[F] 5

Answer: [C], [D], and [E]

If $x^2 - 25 < 0$, then $x^2 < 25$. Since a negative x squares to a positive value, x can be negative as long as the square doesn't exceed 25. This implies that $-5 < x < 5$. Note especially that x cannot equal either 5 or -5, as the square would be equal to 25 rather than less than 25. These limits rule out [A], [B], and [F].

What is the decimal equivalent of $\frac{5}{8}$?

Answer: 0.625

It is a good idea to memorize the decimal equivalents for the various eighths fractions, or at least remember that $\frac{1}{8}=0.125$. With that, you can reason that $\frac{5}{8}=\frac{1}{2}+\frac{1}{8}=0.5+0.125=0.625$

What is the percent equivalent of 2.25?

Answer: 225%

To convert from a decimal to a percent, move the decimal point two places to the right and add a percent symbol: $2.25 \longrightarrow 225\%$.

If you forget that method, you could reason this way:

1 thing is "a whole thing," or 100% of a thing.

0.25 of something is $\frac{1}{4}$ or 25% of something.

So $2.25 = 1 + 1 + 0.25 = 100\% + 100\% + 25\% = 225\%$

What is the decimal equivalent of $\frac{81}{72}$?

Answer: 1.125

First, reduce the fraction and convert to a mixed number:

$$\frac{81}{72}=\frac{9}{8}=1\tfrac{1}{8}$$

It is a good idea to memorize that $\frac{1}{8}=0.125$. So, $1\tfrac{1}{8}=1+0.125=1.125$

What is 82% as a fully reduced fraction?

Answer: $\frac{41}{50}$

To convert from a percent to a fraction, use the digits of the percent for the numerator and 100 for the denominator. Then simplify:

$$82\% = \frac{82}{100} = \frac{41}{50}$$

If z is 37.5% of y, what fraction of z is y?

Answer: $\frac{8}{3}$

Translate "z is 37.5% of y" to equation form: $z = 37.5\%$ of y, or the equivalent, $z = 0.375y$.

Translate "what fraction of z is y?" as $\frac{?}{?}z = y$. So, y needs to be isolated to answer this question.

If $z = 0.375y$, then $y = \frac{z}{0.375}$. The decimal is easier to deal with in fraction form:

$y = \frac{z}{3/8} = z \times \frac{8}{3}$. The fraction we are looking for is $\frac{8}{3}$.

One quarter of the light bulbs on a string of lights were burned out. If 14 burned-out bulbs are replaced with working bulbs, 85% of the lights on the string now work. How many light bulbs total are on the string?

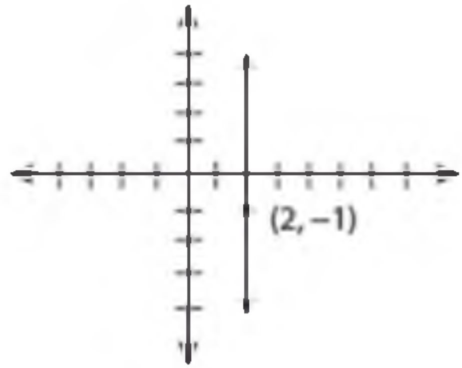
Answer: 140

If one quarter of the light bulbs on a string of lights were burned out, three quarters, or 75%, of the bulbs were working.

The 14 changed bulbs increased the percent of working bulbs by 85% – 75% = 10%. Thus, 10% of the total bulbs = 14, or there are 140 bulbs total.

In math form:

$$\begin{aligned} 0.75x + 14 &= 0.85x \\ 14 &= 0.1x \\ x &= 140 \end{aligned}$$

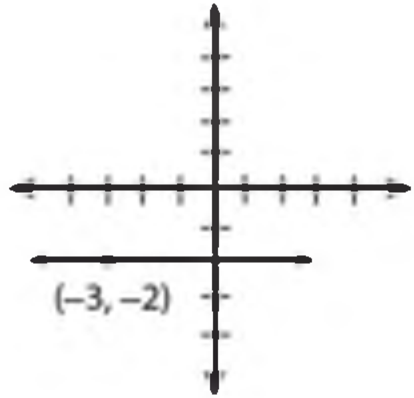


What is the equation of the line shown?

Answer: $x = 2$

Because the line is vertical, the x value is always 2, regardless of the value of y. Because x does not vary with y, the equation does not need to specify y. You can read "$x = 2$" to mean "x must equal 2, and y can be anything."

The equation of a vertical line cannot be written in slope-intercept form, because the slope is undefined in this case.



What is the equation of the line shown?

Answer: $y = -2$

Because the line is horizontal, the y value is always -2, regardless of the value of x. Because y does not vary with x, the equation does not need to specify x. You can read "$y = -2$" to mean "y must equal -2, and x can be anything."

Or, consider that this equation is in slope-intercept form: $y = -2$ is equivalent to $y = 0x - 2$. The y-intercept is -2 and the slope is 0 for a horizontal line.

Working together at the same constant rate, Stanley and Mimi can plant 6 trees in 6 hours. How long would it take Mimi, working alone, to plant 4 trees?

Answer: 8 hours

Together, Stanley and Mimi plant $\frac{6 \text{ trees}}{6 \text{ hours}} = 1$ tree per hour.

Since they work at the same rate, each could plant half as fast as this: $\frac{1}{2}$ tree per hour.

To plant 4 trees, Mimi would need $\frac{4 \text{ trees}}{\frac{1}{2} \text{ trees/hour}} = 4 \times \frac{2}{1} = 8$ hours.

Light travels at an approximate speed of 299,792,000 meters per second. Approximately how many minutes does it take for light to travel 95 million kilometers, where 1 kilometer = 1,000 meters?

Round your answer to the nearest minute.

431

Answer: 5 minutes

The speed of light is given in units of *meters per second*. The distance is given in *kilometers*, and the question asks for the time in *minutes*.

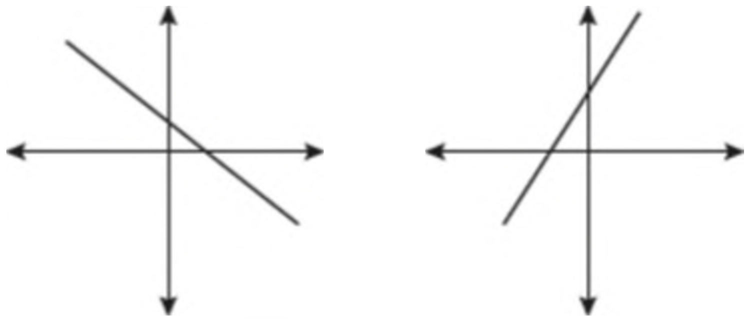
First, approximate the speed and convert to kilometers per second:

$$\frac{\text{approx. } 300{,}000{,}000 \text{ meters}}{\text{second}} \times \frac{\text{kilometer}}{1{,}000 \text{ meters}} \times \frac{60 \text{ second}}{\text{minute}} = \frac{18{,}000{,}000 \text{ kilometers}}{\text{minute}}$$

Now, using the calculator, solve for time. Again, keep track on paper of the units and the steps you are taking.

$$\frac{95 \text{ million kilometers}}{18 \text{ million kilometers/minute}} \approx 5.3 \text{ minutes}$$

Estimate the slope of the lines below:



Note: Figures are drawn to scale.

Answer: Approximately -1 (left figure) and 2 (right figure).

In the left figure, the line slopes down from left to right, so the slope is negative. The angle of the line is approximately 45° from both the x and y axes. Also, note that the y-intercept and x-intercept are about the same distance from the origin. In other words, $|\text{rise}| = |\text{run}|$, or $|\text{slope}| = \frac{|\text{rise}|}{|\text{run}|} = 1$. The slope is thus approximately -1.

In the right figure, the line slopes up from left to right, so the slope is positive. The angle of the line is somewhat steeper than 45° from the x axis. So, the slope > 1. Also, note that the y-intercept is about twice as far as the x-intercept from the origin. In other words, $|\text{rise}| = 2 \times |\text{run}|$, or $|\text{slope}| = \frac{|\text{rise}|}{|\text{run}|} = 2$.
The slope is thus approximately 2.

A bag holds 2 red marbles and 2 blue marbles. Two marbles will be selected in succession and without replacement in the bag. What is the probability of first drawing a blue marble, then drawing a red marble?

Answer: 1/3

On the first selection, the chance of selecting a blue marble first is $\frac{2}{4}$.

On the second selection, there are only 3 marbles remaining in the bag, of which 2 would be red (assuming blue selected on the first draw). Chance of selecting a red marble second is $\frac{2}{3}$.

Chance of first selecting blue AND second selecting red is $\frac{2}{4}\times\frac{2}{3}=\frac{1}{3}$.

A bag holds 2 red marbles and 2 blue marbles. When selecting two marbles from the bag at once (without replacement), what is the probability of selecting one of each color?

Answer: $\frac{2}{3}$

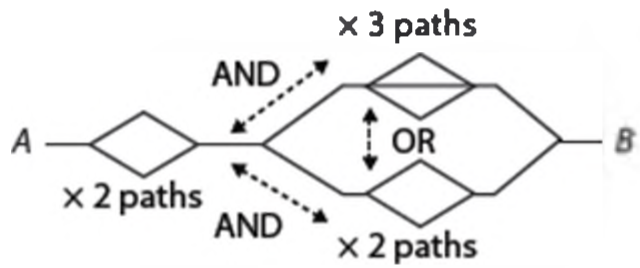
Ignoring the color of the marbles for a moment, there are $\frac{4!}{2!2!}=\frac{(4)(3)(2)(1)}{(2)(1)(2)(1)}=\frac{(4)(3)}{(2)}=6$ ways to select two marbles.
There are only 2 ways to select a pair that is <u>not</u> "one of each color":

- Select 2 red and leave 2 blue in the bag
- Select 2 blue and leave 2 red in the bag

This implies that there are $6 - 2 = 4$ ways to select a mixed-color pair. Probability of selecting one marble of each color = $\frac{4}{6}=\frac{2}{3}$.

A toy boat floats on a river that branches as indicated in the figure below. If the river flows only from left to right at all points, how many different paths from *A* to *B* are possible for the toy boat?

Answer: 10



There are $2 \times (3 + 2) = 2 \times 5 = 10$ possible paths.

The boat takes a path through the left-most node AND the top right node, OR the boat takes a path through the left-most node AND the bottom right node. There is no way for the boat to pass through both the top AND bottom right nodes—not without backtracking and going against the river current. Thus, the decision at the right nodes is an "OR" situation, requiring addition. The right part of the diagram is equivalent to 5 simple paths.

Simplify:

$$4^{12} + 4^{11} + 4^{10} - 5(4^{10})$$

Answer: 4^{12}

Factor 4^{10} out of each term and simplify:

$$\begin{aligned}4^{12}+4^{11}+4^{10}-5\left(4^{10}\right) &= 4^{10}\left[4^{2}+4^{1}+4^{0}-5\right]\\ &= 4^{10}[16+4+1-5]\\ &= 4^{10}[16]=4^{10}\left[4^{2}\right]=4^{12}\end{aligned}$$

Or, simplify the last two terms first:

$$\begin{aligned}4^{12}+4^{11}+\left[4^{10}-5\left(4^{10}\right)\right] &= 4^{12}+4^{11}+\left[1\left(4^{10}\right)-5\left(4^{10}\right)\right]\\ &= 4^{12}+4^{11}+\left[-4\left(4^{10}\right)\right]\\ &= 4^{12}+4^{11}+\left[-\left(4^{11}\right)\right]=4^{12}\end{aligned}$$

Frequency Distribution for List *L*

Number	2	4	6	8
Frequency	5	16	8	13

List *L* contains 42 numbers.

<u>Quantity A</u>	<u>Quantity B</u>
The median of list *L*	The average of list *L*

Answer: (B) Quantity B is greater.

In a list of 42 numbers, the median is the average of the two middle terms, the 21^{st} and the 22^{nd}. The frequency distribution shows that 5 + 16 = 21 numbers are either 2 or 4 (low) and 8 + 13 = 21 numbers are either 6 or 8 (high). The median is $\frac{4+6}{2}=5$.

Don't calculate the average of the list. The average would equal 5 if the average of the "low" half of the numbers were 3 and the average of the "high" half of the numbers were 7. However, both the low numbers and the high numbers are skewed toward the high side: there are more 4's in the list than 2's, and more 8's than 6's. The average is greater than 5.

If $x \neq 0$, simplify:

$$\frac{x^{-3}\left(x^{4}\right)}{\left(x^{-2}\right)^{3}}$$

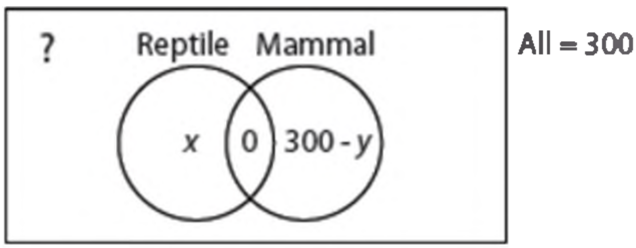
Answer: x^7

$$\frac{x^{-3}\left(x^{4}\right)}{\left(x^{-2}\right)^{3}}=\frac{x^{-3+4}}{x^{-2\times 3}}=\frac{x^{1}}{x^{-6}}=x^{1-(-6)}=x^{1+6}=x^{7}$$

Among the 300 animals at a zoo, there are x reptiles and y animals that are not mammals. (No reptiles are mammals, and no mammals are reptiles.) In terms of x and y, how many animals are neither mammals nor reptiles?

Answer: $y - x$

Even though there is no overlap between mammals and reptiles, a Venn Diagram is helpful. Put 0 in the overlap portion. Every animal outside the mammal circle is given as y, so the remainder inside the mammal circle is $300 - y$.



Sum all the areas of the diagram, and solve for the ? mark.

$? + x + 0 + (300 - y) = 300$

$? + x - y = 0$

$? = y - x$

If $\frac{-a+4b}{3a-2b}=-1$, and if $a \neq \frac{2}{3}b$ which of the following statements must be true? Select <u>all</u> that apply.

[A] a is the additive inverse of b.

[B] a is greater than b.

[C] a is less than b.

[D] a is 3 times b.

[E] $3a$ does not equal $2b$.

Answer: [A] and [E]

$$\frac{-a+4b}{3a-2b}=-1, \text{ so } \begin{cases} -a+4b=-1(3a-2b) \\ -a+4b=2b-3a \\ 2b=-2a \\ b=-a \end{cases}$$

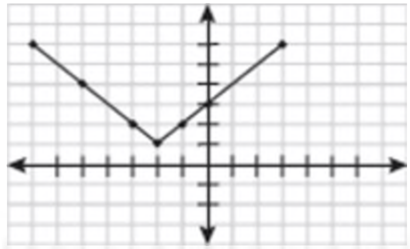
[A] TRUE. Additive inverses sum to 0: $a + b = a + (-a) = 0$.

[B] MAYBE. a is greater than b if a is positive. (e.g. $a = +2$ and $b = -2$)

[C] MAYBE. a is less than b if a is negative. (e.g. $a = -5$ and $b = +5$)

[D] FALSE. a is actually -1 times b. If a or b could equal zero, a would equal $3b$ as well, but since $a \neq \frac{2}{3}b$, neither variable can equal zero.

[E] TRUE. If $3a = 2b$, the original fraction would be undefined, but it was actually equal -1, so $3a$ must not be equal $2b$.



The figure above shows the graph of the function f in the xy-plane. What is the value of $f\left(\frac{f(-3)}{2}\right)$?

Answer: 4

For multiple or "nested" functions, start from the innermost function first: $f(-3)$. Look up $x = -3$ on the graph, and determine the y value there. $(-3, 2)$ is a point on the graph of function f, or $f(-3) = 2$.

Now, the next function: $f\left(\frac{f(-3)}{2}\right)=f\left(\frac{2}{2}\right)=f(1)$ Look up $x = 1$ on the graph, and determine the y value there. $(1, 4)$ is a point on the graph of function f, or $f(1) = 4$.

The cost of a t-shirt is $5, but bulk orders of at least 120 t-shirts are discounted 10%.

Quantity A	Quantity B
The number of t-shirts purchased for $540	125

Answer: (B) Quantity B is greater.

"Cheat" off of the easier quantity. If 125 t-shirts are purchased, the cost would be $(125)(\$5) = \$625$, minus a 10% discount of \$62.50, bringing the cost to \$562.50.

\$540 is less than \$562.50, so even with the bulk order discount, \$540 would purchase fewer shirts.

Translate into algebra form:

Jeremiah is 15 years older than Michelle, who is half as old as Frank.

Answer: $J = M + 15$ and $\left[M=\frac{1}{2}F\right]$ or equivalent.

Test: "Jeremiah is 15 years older than Michelle" so Michelle could be 10 and Jeremiah could be 25. It is true that $25 = 10 + 15$.

Test: "...Michelle, who is half as old as Frank." so Michelle could be 10 and Frank could be 20. It is true that $10=\frac{1}{2}(20)$. .

Sketch pads cost \$3 and charcoal pencils cost \$1. If each student in an art class needs 2 pencils and a sketch pad, \$93 will buy a full set of supplies for how many students?

Answer: 18

Each student in an art class needs 2 pencils [cost = 2($1) = $2] and a sketch pad [cost = $3], supplies for each student cost a total of $5.

$93 divided by $5 is 18, with $3 remaining. Since the $3 remaining will not by enough supplies for a 19th student, $93 will buy a full set of supplies for 18 students.

Translate into algebra form and simplify:

A is *B* percent of 2 times *C*.

445

Answer: $A=\frac{BC}{50}$ or the equivalent $50A = BC$

"A is": $A =$

"B percent": $\frac{B}{100}$

{Test a number if you need to check: 11 percent $=0.11=\frac{11}{100}$}

"of": $\times$

"2 times C": $2C$

Putting it together: $A=\frac{B}{100}\times 2C=\frac{2BC}{100}=\frac{BC}{50}$

Translate into algebra form:

There are 12 fewer members of Club A than members of Club B and Club C combined.

Answer: $A = B + C - 12$ or any equivalent

"12 fewer" clearly means $- 12$, but it can be tricky to determine what to subtract 12 from. Don't assume $A - 12$ is part of the equation just because 12 is closest to Club A in the sentence! Look for the full comparison phrase "12 fewer...than": subtract 12 from what follows the "than," which is "Club B and Club C combined," so $B + C - 12$.

Test numbers to check. If $B = 10$ and $C = 20$, Clubs B and C combined have 30 members. Club A has 12 fewer than that, or $A = 18$. Plug into $A = B + C - 12$, getting $18 = 10 + 20 - 12$, which is true.

Machine A can produce 100 widgets in 3 hours.

Machine B can produce 100 widgets in 2 hours.

Quantity A	Quantity B
The widget production rate of Machine A	The widget production rate of Machine B

Answer: (B) Quantity B is greater.

The machines produce the same number of widgets, but Machine B does so in less time, so its production rate must be greater.

$R \times T = W$, so when work is constant, rate and time are inversely related: more time = lower rate, and vice versa.

Specifically, A's rate is $\frac{100}{3} = 33.3$ widgets per hour, while B's rate $\frac{100}{2} = 50$ widgets per hour.

Traveling at their constant respective rates, Jillian runs twice as fast as Bob walks.

Quantity A	**Quantity B**
The time it takes Jillian to run 1,000 meters	The time it takes Bob to walk 600 meters

Answer: (B) Quantity B is greater.

$R \times T = D$, so if time were equal, rate and distance would be proportional.

If Bob walks half as fast as Jillian, he would cover half the distance Jillian does in the same time. Bob would walk 500 meters in the time it takes Jillian to run 1,000 meters. Bob walked farther than 500 meters, so he walked for more time than that. Quantity B is greater than Quantity A.

The population of bacteria in a dish doubles every 10 seconds.

Quantity A	Quantity B
80 times the population of bacteria in the dish 1 minute ago.	The population of bacteria in the dish now.

Answer: (A) Quantity A is greater.

Between 1 minute ago and now, the population of bacteria doubled $\frac{60\text{sec}}{10\text{sec}} = 6$ times. The population of bacteria in the dish now is $2^6 = 64$ times the population of bacteria in the dish 1 minute ago.

80 times the population 1 minute ago is greater than 64 times that same population.

In their respective vehicles and at their respective constant rates, a police officer is pursuing a bank robber. The bank robber is currently half a mile ahead, but the police officer is traveling 10 miles per hour faster. How long from now will the police officer catch up to the bank robber?

450

Answer: 3 minutes

For a "chase" scenario, you do not need to know the respective rates for the police officer and bank robber—the *relative* rate is all that is needed. Alter the usual $R \times T = D$ equation to this:

$(R_{\text{cop}} - R_{\text{robber}}) \times T =$ change in the gap between the cars

10 miles per hour $\times T = \frac{1}{2}$ mile

$$T = \frac{\frac{1}{2}\text{mile}}{10\text{ miles per hour}}$$

$$T = \frac{1}{20}\text{hour} \times 60\text{ minutes/hour} = 3\text{ minutes}$$

A water reservoir empties at a constant rate of 520,000 gallons per day, but is refilled by periodic rain storms. On average, if it rains every 5 days, how many gallons per rainstorm are required to replace the used water?

Answer: 2.6 million

On average, if it rains every 5 days, each storm must replace the water used over a period of 5 days.

520,000 gallons per day × 5 days = 2,600,000 gallons = 2.6 million gallons

At an ice cream shop, customers can order a single scoop of chocolate, vanilla, or strawberry ice cream in either a cone or a cup. If candy sprinkles are optional, how many different orders are possible?

Answer: 12

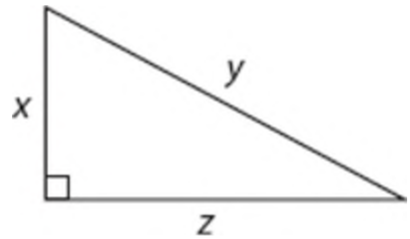
Choice of flavors: 3 options (chocolate, vanilla, or strawberry)

Choice of vessel: 2 options (cone or cup)

Choice of sprinkles: 2 options (yes or no)

Customers can order flavor AND vessel AND sprinkle option. “And” means multiply: $3 \times 2 \times 2 = 12$ order combinations.

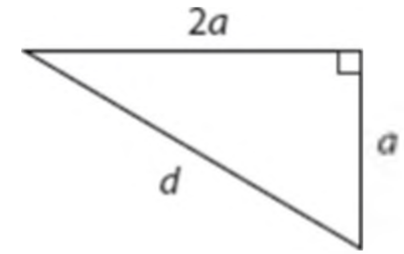
What is x in terms of y and z?



Answer: $x=\sqrt{y^2-z^2}$

By Pythagorean Theorem, $x^2 + z^2 = y^2$, so $x^2 = y^2 - z^2$, and therefore $x=\sqrt{y^2-z^2}$.

What is d in terms of a?



Answer: $d = a\sqrt{5}$

By Pythagorean Theorem, $a^2 + (2a)^2 = d^2$. Remember to square both the 2 and the a in the parentheses! So, $a^2 + 4a^2 = d^2$. Simplifying, $5a^2 = d^2$ and $d = \sqrt{5a^2}$. Therefore, $d = a\sqrt{5}$.

Quantity A

$$\frac{\frac{3}{\sqrt{2}}}{\frac{2}{\sqrt{3}}}$$

Quantity B

1

Answer: (A) Quantity A is greater.

$$\frac{\frac{3}{\sqrt{2}}}{\frac{2}{\sqrt{3}}}=\frac{3}{\sqrt{2}}\times\frac{\sqrt{3}}{2}=\frac{3}{2}\times\frac{\sqrt{3}}{\sqrt{2}}=\frac{3}{2}\times\sqrt{\frac{3}{2}}$$

Both $\frac{3}{2}$ and $\sqrt{\frac{3}{2}}$ are greater than 1, so their product will also be greater than 1.

Simplify:

$$\frac{\sqrt{\pi^4}}{\pi}$$

456

Answer: π

$$\frac{\sqrt{\pi^4}}{\pi}=\frac{\sqrt{\left(\pi^2\right)^2}}{\pi}=\frac{\left(\pi^2\right)}{\pi^1}=\pi^{2-1}=\pi^1$$

What is 45.57 as a fraction?

Answer: $\frac{4,557}{100}$

The decimal goes to two places: the first is the tenths $\left(\frac{1}{10}\right)$ place, the second is the hundredths $\left(\frac{1}{100}\right)$ place. Put the entire number (with no decimal) over 100.

In the decimal form of $\frac{37}{99}$, how many terms are in the repeating cycle of digits to the right of the decimal point?

458

Answer: Two

The decimal form is $0.37373737... = 0.\overline{37}$. There are two terms (3 and 7) in the repeating cycle of digits after the decimal point.

Any two-digit number over 99 repeats in this simple pattern. Three-digit numbers overs 999 repeat the same way, and so on.

$$ab \neq 0$$

$$|a + b| = |a| + |b|$$

<u>Quantity A</u>	<u>Quantity B</u>
ab	0

Answer: (A) Quantity A is greater.

Since $ab \neq 0$, neither variable can be zero. Each is either positive or negative.

If $|a + b| = |a| + |b|$, then a and b must have the same sign. If they are both positive, the absolute value bars don't do anything, and can be removed. It is true that $a + b = a + b$. If a and b are both negative, the absolute value bars mean "multiply by -1 to turn the negative into a positive" and the equation becomes $-(a + b) = -a + (-b)$, which is also true. However, if only one of the variables is negative, $|a + b| < |a| + |b|$. For example, $|5 + (-3)| < |5| + |-3|$, since $2 < 5 + 3$.

Quantity A: ab = (pos)(pos) or (neg)(neg) = positive in either case

102 is what fraction of 17?

460

Answer: 6, or $\frac{6}{1}$

Write the question as an equation: 102 = what fraction? × 17, so what fraction? $= \frac{102}{17} = \frac{6}{1}$. It may seem weird that the "fraction" is an integer, but all numbers can be written as fractions. This fraction just happens to reduce to an integer.

102 is 6 times 17, so it is $\frac{6}{1}$ of 17.

4 is 5% of what number?

461

Answer: 80

Write the question as an equation: $4 = \frac{5}{100} \times$ what number?

So, $\frac{100}{5} \times 4 =$ what number? The answer is 80. Alternatively, 5% or $\frac{1}{20}$ of some number is 4. Thus 4(20) = 80 = the number.

Annual number of visitors to a museum

Year	Percent change from previous year
2010	−5%
2011	+10%
2012	−2%

What was the cumulative percent change in annual visitors to the museum from 2009 to 2011?

Answer: 4.5%

Use a Smart Number of 1,000.

If 1,000 people visited the museum in 2009, then 5% fewer, or 950 people, visited in 2010. In 2011, 10% more than 950 people visited the museum: $950 + (0.10)(950) = 950 + 95 = 1{,}045$ people visited in 2011.

The number of visitors increased from 1,000 in 2009 to 1,045. This is an increase of 45 per 1000, or 4.5 per 100, which is 4.5%.

Read carefully! We can ignore the change in 2012.

At what point on the coordinate plane will the lines $0 = x - 2$ and $y = 4$ intersect?

Answer: (2, 4)

$0 = x - 2$ is equivalent to $x = 2$. This is a vertical line through +2 on the x-axis. Likewise, $y = 4$ is a horizontal line through +4 on the y-axis. These lines intersect at (2, 4).

Alternatively, if $x = 2$ and $y = 4$, then the point (x, y) must be (2, 4).

Given $x > 0$, simplify:

$$\sqrt{\frac{x^{a-b}}{x^{b-a}}}$$

Answer: $x^{a\text{-}b}$

$$\sqrt{\frac{x^{a-b}}{x^{b-a}}}=\sqrt{\frac{x^{a-b}}{x^{-(a-b)}}}=\sqrt{x^{a-b}\times x^{+(a-b)}}=x^{a-b}$$

Alternatively,

$$\sqrt{\frac{x^{a-b}}{x^{b-a}}}=\sqrt{x^{(a-b)-(b-a)}}=\sqrt{x^{a-b-b+a}}=\sqrt{x^{2a-2b}}=\sqrt{\left(x^{a-b}\right)^2}=x^{a-b}$$

As long as $x > 0$, $x^{a\text{-}b}$ is positive, so it equals the square root of its square. (That's *not* true for negative numbers.)

Solve for x and y:

$$3x + 5y = 9$$

$$2x + 5y = 11$$

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Answer: $x = -2$ and $y = 3$

Because the coefficient on the y term is the same in both equations, elimination (of the y terms) is easier than substitution. Subtract the whole 2nd equation from the 1st:

$$\begin{array}{r} 3x + 5y = 9 \\ \underline{-(2x + 5y = 11)} \\ x = \quad -2 \end{array}$$

Plug $x = -2$ into either equation:

$3(-2) + 5y = 9$

$-6 + 5y = 9$

$5y = 15$

$y = 3$

What is the domain of the function

$$f(x) = \frac{x-3}{x+2}?$$

Answer: All real numbers except −2.

The domain of a function is the set of all permissible inputs (here, the *x* values). The fraction $\frac{x-3}{x+2}$ is defined for all real numbers except $x = -2$. Fractions with zero as the denominator are undefined.

If $1,000 is invested at a simple annual interest rate of 1.5%, what is the value of the investment after 4 years?

Answer: \$1,060

The formula for simple interest is $V=P\left(1+\frac{rt}{100}\right)$, where P is the principal amount invested, r is the annual interest rate, and t is the number of years.

For this question:

$$V=\$1{,}000\left(1+\frac{(1.5)(4)}{100}\right)=\$1{,}000\left(1+\frac{6}{100}\right)$$
$$=\$1{,}000(1+0.06)=\$1{,}000(1.06)=\$1{,}060$$

In which year was the number of visitors to the museum <u>least</u>?

Annual number of visitors to a museum

Year	Percent change from previous year
2010	−5%
2011	+10%
2012	−2%

(A) 2009

(B) 2010

(C) 2011

(D) 2012

Answer: (B) 2010

There were 5% fewer visitors to the museum in 2010 than 2009, so 2009 cannot be the year with the least visitors. Likewise, there were 10% more visitors to the museum in 2011 than 2010, so 2011 cannot be the year with the least visitors. Eliminate (A) and (C).

To compare 2010 and 2012, use a Smart Number of 1,000 for the number of visitors in 2010.

Year	Percent change from previous year	Number of visitors (Smart Number)
2010	−5%	1,000
2011	+10% (= +100)	1,100
2012	−2% (= −22)	1,078

There were more visitors in 2012, so 2010 was the year with the fewest visitors.

What is the quadratic formula, and what is its purpose ?

Answer: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, where a, b, and c are real numbers and $a \neq 0$ in a quadratic equation of the form $ax^2 + bx + c = 0$. Plugging a, b, and c into the equation yields the solution(s) of the quadratic equation.

Often, we can find the solutions in other ways (e.g., by factoring), but occasionally the quadratic formula is the only option.

For what values of x is y a real number?

$$y = 2\sqrt{x+3} + x - 4$$

Answer: $x \geq -3$

y is a real number when $2\sqrt{x+3}+x-4$ is a real number. The $+x - 4$ part of the expression is a real number for all real x values. But $\sqrt{x+3}$ is only a real number when $x + 3 \geq 0$, or $x \geq -3$.

If \$200 is invested at an annual interest rate of 4%, compounded quarterly, what will the value of the investment be at the end of 6 months?

Answer: $204.02

For compound interest, $V = P\left(1+\frac{r}{100n}\right)^{nt}$, where *V* is the value of the investment at the end of *t* years if *P* is the amount invested at an annual interest rate of *r* percent and compounds *n* times per year.

$$V = 200\left(1+\frac{4}{100\times 4}\right)^{4(0.5)} = 200\left(1+\frac{1}{100}\right)^{2} = 200(1.01)^{2} = 204.02\,.$$

Alternatively, think "4% annually but compounded quarterly is like increasing 1% every 3 months." In 6 months, two such 1% increases occur:

200 + 2.00 = 202

202 + 2.02 = 204.02

If \$1,000 is invested at a simple annual interest rate of 10%, what will the value of the investment be at the end of 2.5 years?

Answer: $1,250

For simple interest, $V = P\left(1 + \frac{rt}{100}\right)$, where V is the value of the investment at the end of t years if P is the amount invested at an annual interest rate of r percent. Here,

$$V = 1{,}000\left(1 + \frac{10 \times 2.5}{100}\right) = 1{,}000\left(1 + \frac{25}{100}\right) = 1{,}000(1.25) = 1{,}250.$$

6 people are traveling on an airplane, each with 2 bags. If the bags could be placed either in the cabin or the cargo area, with no restrictions, how many possibilities are there for the number of bags in the cabin?

Answer: 13

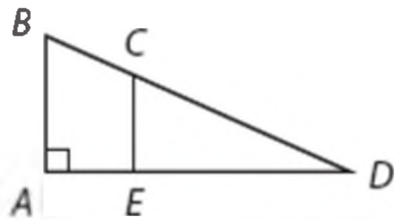
Watch out! This is not a combinations question about how many different arrangements are possible for the bags (for which it would matter whose bags are in the cabin and whose are in cargo, for example). This is simply: How many answers are possible for the question, "What is the number of bags in the cabin?"

Each person could carry 0, 1, or 2 bags into the cabin. The maximum number of bags in the cabin is (2 bags/person)(6 people) = 12 bags. The minimum number of bags in the cabin is (0 bags/person)(6 people) = 0 bags. The possibilities are 0, 1, 2,..., 11, 12 bags. There are 13 possibilities for the number of bags in the cabin.

If $g(x)=\frac{|3-2x|}{-x}$, what is g(−10)?

Answer: 2.3

$$g(-10)=\frac{|3-2(-10)|}{-(-10)}=\frac{|3-(-20)|}{10}=\frac{|3+20|}{10}=\frac{23}{10}=2.3$$



$AB \parallel CE$ and $CE = \frac{2}{3}AB$

If the area of triangle ABD is 54, what is the area of triangle CDE?

Answer: 24

Line segment *CE*, drawn inside triangle *ABD* and parallel to *AB*, creates a similar triangle *CDE*. The height of triangle *CDE* is 2/3 the height of triangle *ABD*. Because the triangles are similar, the base of triangle *CDE* is *also* 2/3 the base of triangle *ABC*.

Area of triangle $ABC = bh = 54$

Area of triangle $CDE = \frac{2}{3}b \times \frac{2}{3}h = \frac{4}{9}bh = \frac{4}{9}(54) = 24$

Which of the following lines are perpendicular to $y = -3x + 4$? Indicate <u>all</u> such lines.

[A] $y = -\frac{1}{3}x - 2$

[B] $y = x + \frac{3}{4}$

[C] $y = \frac{1}{3}x - 6$

[D] $3y = x + 4$

Answer: [C] and [D]

Lines are perpendicular when their slopes are negative reciprocals of one another. That is, the product of the two slopes equals –1.

The slope of $y = -3x + 4$ is -3. The negative reciprocal of -3 is $\frac{1}{3}$.
[C] $y=\frac{1}{3}x-6$ is already in slope-intercept form. Slope = $\frac{1}{3}$.

[D] $3y = x + 4$ is equivalent to $y=\frac{1}{3}x+\frac{4}{3}$, which has slope = $\frac{1}{3}$.

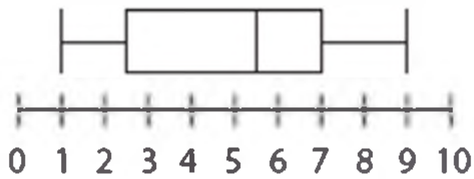
List X consists of 100 unique numbers.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
The range of list X.	The interquartile range of list X.

Answer: (A) Quantity A is greater.

The range of any list is the difference between the greatest term and the least term. The interquartile range of any list is the difference between the third quartile Q_3 and the first quartile Q_1.

Because all of the numbers in list X are unique, i.e. no repeats in the list, Q_3 < greatest term and Q_1 > least term. Thus, $Q_3 - Q_1$ < greatest term – least term.



Which one of the following lists is summarized by the box-and-whisker plot above?

(A) 1, 4, 5, 6, 6, 7, 7, 8

(B) 4, 4, 4, 5, 5, 5, 6, 9

(C) 1, 2, 3, 5, 6, 7, 7, 9

Answer: (C)

A box-and whisker plot shows the full range of the data (i.e. the least and greatest term in the set) with the outer-most lines, or whiskers. Here, minimum = 1 and maximum = 9. That is enough to select (C).

Also, the boxes in the box-and-whisker plot indicate the 2nd and 3rd quartiles, so the line between the boxes is the median. In a set with 8 terms, each quartile contains 2 terms, and the median is the average of the 4th and 5th largest terms. For (C), that median is 5.5.

Incidentally, the left edge of the boxes is Q_1, the average of the greatest term in the 1st quartile and the smallest term in the 2nd quartile, or 2.5 for (C). The right edge of the boxes is Q_3, the average of the greatest term in the 3rd quartile and the smallest term in the 4th quartile, or 7 for (C).

Quantity A	Quantity B
$\frac{4}{7}$	$\frac{2}{3}$

Answer: (B) Quantity B is greater.

There are several ways to answer. Use the calculator:

$\frac{4}{7}=0.5714$ ...and so $\frac{2}{3}=0.6666$ ...is greater.

Or note that $\frac{2}{3}=\frac{4}{6}$ and $\frac{4}{6}>\frac{4}{7}.$, as a larger denominator means a smaller fraction.

Finally, we can cross-multiply upwards:

$12\ \frac{4}{7} \times \frac{2}{3}\ 14$ 12 < 14, so $\frac{2}{3}$ is greater.

Quantity A	Quantity B
$\frac{89}{91}$	$\frac{35}{37}$

Answer: (A) Quantity A is greater.

There are several ways to answer. Use the calculator:

$\frac{89}{91} = 0.9780$ ...and $\frac{35}{37} = 0.9459$ .... so $\frac{89}{91}$ is greater.

Or, note that each numerator is 2 less than its respective denominator. $\frac{2}{91} < \frac{2}{37}$, since a larger denominator means a smaller fraction. So Quantity A is 1 – a smaller amount and Quantity B is 1 – a larger amount. Quantity A is greater.

If $k(x)=\frac{|-x|}{x}$ for all non-zero x, what is the value of $k(20) - k(-9)$?

Answer: 2

$$k(20)=\frac{|-20|}{20}=\frac{20}{20}=1$$

$$k(-9)=\frac{|-(-9)|}{-9}=\frac{|9|}{-9}=\frac{9}{-9}=-1$$

$$k(20)-k(-9)=1-(-1)=1+1=2$$

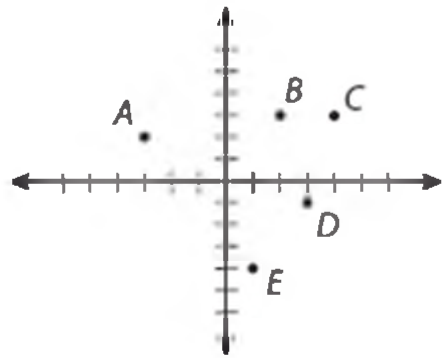
Simplify:

$$3(y - x) - 5(2x - y)$$

Answer: $8y - 13x$

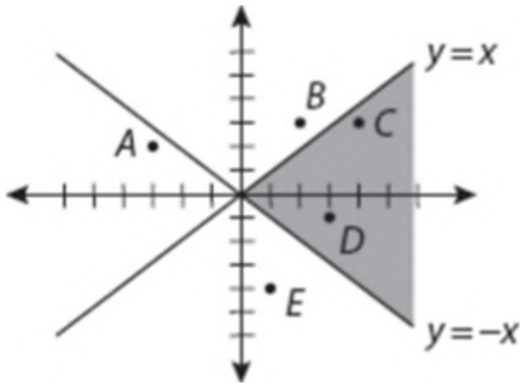
$$\begin{aligned} 3(y-x)-5(2x-y) &= 3y-3x-10x+5y \\ &= 3y+5y-3x-10x \\ &= 8y-13x \end{aligned}$$

If $y < x$ and $y > -x$, which of the points on the coordinate plane below are solutions? Indicate all such points.



Answer: C and D

The solutions lie under the line $y = x$ (because $y < x$) and above the line $y = -x$ (be-
cause $y > -x$), in the shaded area below. Only points C and D are in this region.



Quantity A	Quantity B
$\sqrt[4]{1.44} - 1.1$	0

Answer: (B) Quantity B is greater.

Since $1.44 = 144 \div 100$, and both 144 and 100 are perfect squares, 1.44 is a perfect square. The $\sqrt[4]{\ }$ symbol just means take the square root twice.

$$\begin{aligned}\sqrt[4]{1.44}-1.1 &= \sqrt{1.2}-1.1\\ &\cong 1.0954-1.1\\ &\cong -0.0046\end{aligned}$$

Alternatively, notice that $(1.1)^2 = 1.21$, so $\sqrt{1.2} < \sqrt{1.21}$ and Quantity A will be negative.

Events A and B are mutually exclusive. If $P(A \text{ or } B) = 0.67$ and $P(B) = 0.11$, what is $P(A)$?

Answer: 0.56

Events A and B are mutually exclusive, which means either A can happen or B can happen, but if one event happens the other cannot happen. In such cases:

$P(A \text{ or } B) = P(A) + P(B)$

$0.67 = P(A) + 0.11$

$P(A) = 0.67 - 0.11$

$P(A) = 0.56$

If $f(x) = 4x^2 + 3x - 2,$, what is $f(-1)$?

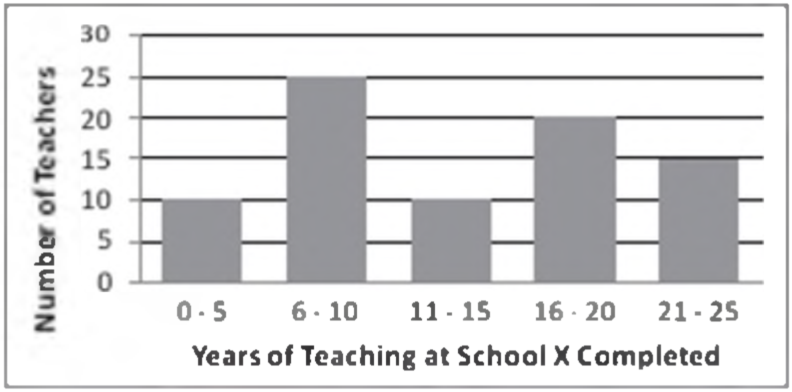
Answer: -1

$$f(x) = 4x^2 + 3x - 2$$

$$f(-1) = 4(-1)^2 + 3(-1) - 2$$

$$f(-1) = 4(1) - 3 - 2$$

$$f(-1) = -1$$

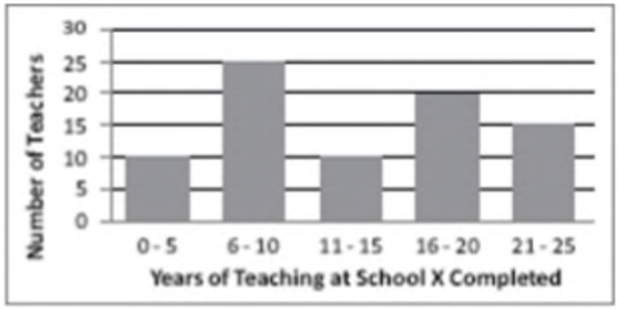


What is the mode for the data above?

Answer: 6–10 years of teaching completed

The mode of any set of data is the measurement that appears most frequently in the set. In this set of data, "Years of Teaching Completed" is the measurement, and "Number of Teachers" indicates the number of times each measurement appears in the set (i.e. how many teachers have that level of teaching experience). The highest bar indicates the mode: 30 teachers have completed 6–10 years of teaching.

Which of the following could be the median of the data set above, in years of teaching completed?



(A) 3

(B) 9

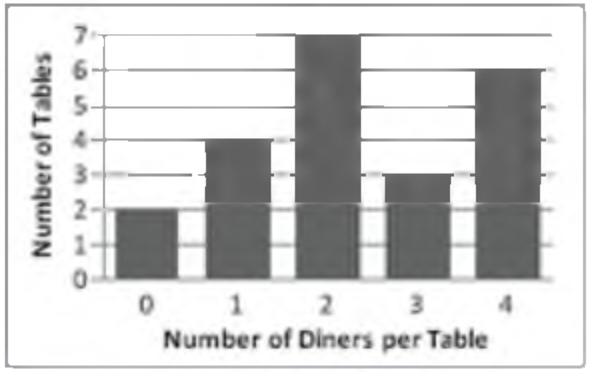
(C) 11

(D) 17

(E) 24

Answer: (C) 11

There are 10 + 25 + 10 + 20 + 15 = 80 teachers represented in the chart. The median "years of teaching completed" is the average for the teachers with the 40^{th} highest and 40^{th} lowest experience levels. Both of these teachers are in the 11–15 years of teaching completed category, so the average could be 11 to 15, inclusive. Only (C) is in this range.



What is the average (arithmetic mean) number of diners per table? Round your answer to the nearest 0.1.

Answer: 2.3

Total number of tables = 2 + 4 + 7 + 3 + 6 = 22

Total number of diners = 2(0) + 4(1) + 7(2) + 3(3) + 6(4) = 0 + 4 + 14 + 9 + 24 = 51

Average number of diners per table = $\frac{51 \text{ diners}}{22 \text{ tables}} = 2.3\overline{18}$ diners per table. Rounded to the nearest 0.1, the answer is 2.3.

Events A and B are independent. If $P(A \text{ or } B) = 0.6$ and $P(A) = 0.2$, what is $P(B)$?

Answer: 0.5

For any two events A and B, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. The last term is subtracted because the probability that *both* events happen is included in both $P(A)$ and $P(B)$.

If A and B are independent events, then $P(A \text{ and } B) = P(A)P(B)$. Substitute and solve:

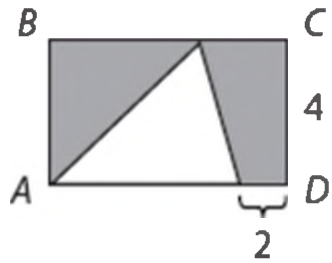
$P(A \text{ or } B) = P(A) + P(A) - P(A)P(B)$

$0.6 = 0.2 + P(B) - 0.2 \times P(B)$

$0.6 - 0.2 = P(B) - 0.2P(B)$

$0.4 = 0.8P(B)$s

$P(B) = 0.4/0.8 = 4/8 = 0.5$

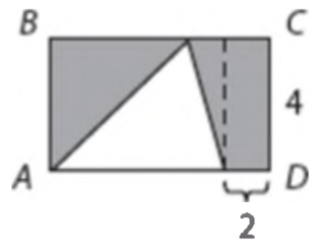


ABCD is a rectangle

In the figure above, what is the area of the shaded region minus the area of the unshaded triangle?

Answer: 8

If not for the 2 by 4 rectangle created by drawing a dotted line as shown, the shaded and unshaded areas would be equal.



The shortened rectangle and the triangle have the same base and height, but the area of the rectangle is base × height, whereas the area of the triangle is $\frac{1}{2}$ base × height. Therefore the triangle is half the area of the rectangle and the shaded area must also equal half the area. Thus, the actual shaded area is bigger only by that 2 by 4 rectangular area: $(2)(4) = 8$.

Which library has the greatest percent of hardcover books?

Answer: Library B

This can be solved visually. Hardcover books are the light gray segment of the bar graph. In Library B, there are clearly more hardcover than paperback books. Thus, the percent of hardcover books in Library B is greater than 50%. The opposite is true in Libraries A and C, which have fewer than 50% hardcover books.

At which library is the percent of books that are paperback closest to 60% of the books?

Answer: Library C

The number of books is not known, as the axis is not labeled. However, the grid lines are evenly spaced, so count them.

At Library C, the number of paperbacks is just over 6 gridline units. The total number of books is around 10 gridline units. 6 of 10 is 60%.

At Library B, paperbacks account for less than half of the books, and at Library A, paperbacks account for more than 80% of the books.

Which library has the most paperback books?

Answer: Library A

This can be solved visually. Paperback books are the black segment of the bar graph. This segment is longest for Library A.

Which of the following functions describes the parabola in the figure shown?

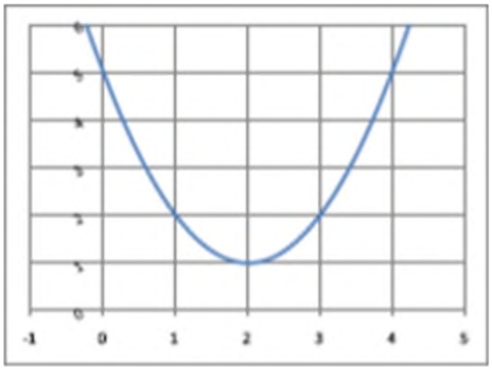
(A) $(x-1)^2+2$

(B) $(x+1)^2-2$

(C) $(x-2)^2+1$

(D) $(x+2)^2-1$

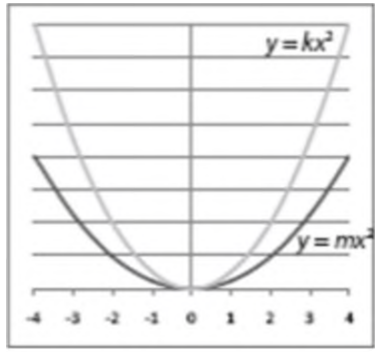
(E) $(x+2)^2+1$



Answer: (C) $(x-2)^2+1$

In general, if the "base" of the U-shaped parabola is at $x = k$, there is a $(x - k)^2$ term in the function. If the U-shape were upside down, the squared term would be negative: $-(x - k)^2$. Also, if the base of the U-shaped parabola is at $y = t$, there is a $+ t$ term in the function. Here, the U is right side up, $k = 2$, and $t = 1$, so $(x - k)^2 + t$ is the function.

Alternatively, note some easy points the function passes through: (2, 1) and (0, 5). Plug into the choices, eliminating any that don't work for these points. Only (C) and (E) work for (0, 5). Of these, only (C) works for (2, 1).



Quantity A	**Quantity B**
k	m

Answer: (A) Quantity A is greater.

In general, for a parabola centered on the y-axis, there is a cx^2 term in the function, where c is a constant. When the parabola is like a right-side-up U, c is positive. When the U is upside down, c is negative. The more narrow the U, the greater the absolute value of c.

Here, both parabolas are right-side-up U shapes, so k and m are both positive. The top/inner parabola is more narrow than the other, so $k > m$.

The function above is defined for all $x \geq -4$. Which of the following equations describes the function shown?

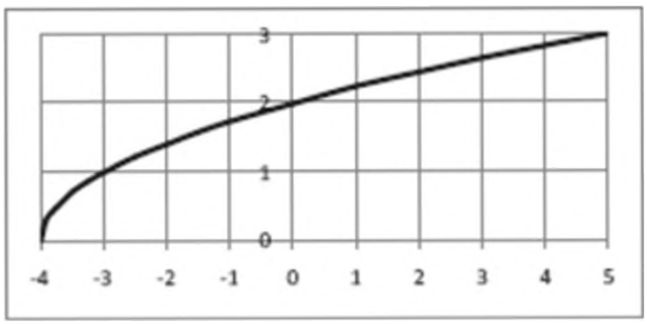
(A) $y = (x + 4)^2$

(B) $y = x^2 + 4$

(C) $y = \sqrt{x} - 4$

(D) $y = \sqrt{x - 4}$

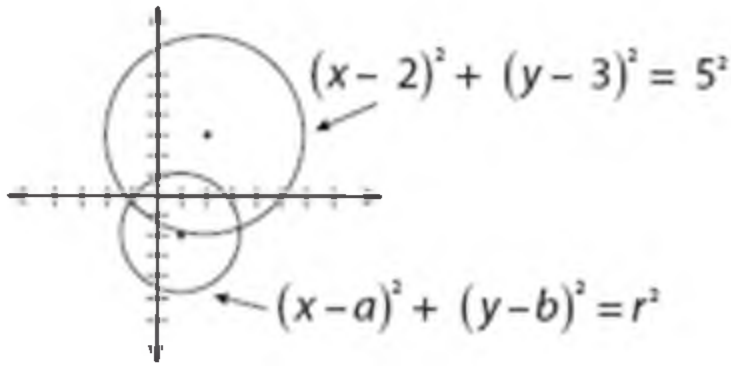
(E) $\sqrt{x + 4}$



Answer: (E)

It helps to know that equations with a variation on the $y = x^2$ form are parabolas centered on a vertical axis (which look like a U or upside-down U), whereas equations with a variation on the $y=\sqrt{x}$ form are parabolas (or half-parabolas, as shown) centered on a horizontal axis. This would rule out (A) and (B). Likewise, the functions in (A) and (B) are defined for all x, not just $x \geq -4$. Equation (C) would be invalid for $x < 0$, and equation (D) would be invalid for $x < 4$, as both would attempt to take the square root of a negative number.

Alternatively, note some easy points the function passes through: (–4, 0) and (0, 2). Plug into the choices, eliminating any that don't work for these points. Only (A) and (E) work for (–4, 0). Only (E) works for (0, 2).



Quantity A	Quantity B
ab	r^2

Answer: (B) Quantity B is greater.

The graph of the equation $(x - a)^2 + (y - b)^2 = r^2$ is a circle with radius r centered at the point (a, b). Even if you didn't know this, you might make an inference from the bigger circle with the given equation. It is centered at (2, 3) and has radius equal 5.

The center of the smaller circle is in quadrant IV, so a is positive and b is negative. Thus, ab is negative. Since r^2 must be positive, Quantity B is greater.

What is the value of $|-1| + |2| + |-3| - |4| - |-5|$?

Answer: -3

Be careful about sign! First, break each number out of absolute value bars, remembering that each negative number will become positive.

$$
\begin{aligned}
|-1|+|2|+|-3|-|4|-|-5| &= (+1)+2+(+3)-4-(+5) \\
&= 1+2+3-4-5 \\
&= -3
\end{aligned}
$$

If $ab < 0$ and $bc > 0$, what is the sign of ac^3?

500

Answer: Negative

If $ab < 0$, then a and b have opposite signs. If $bc > 0$, then b and c have the same sign. This implies that a and c have opposite signs, so ac is negative.

$ac^3 = (ac)(c^2)$. Notice that c^2 must be positive, regardless of the sign of c.

Thus, $ac^3 = (\text{neg})(\text{pos}) = \text{negative}$.